Risk Transfer:

through actuarial approach to reinsurance and it's pricing

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Reinsurance Pricing

1 Introduction and types of reinsurance

2 Exposure Rating Method

3 Burning Cost Rating Method

4 Pareto Method

5 Alternatives of Pareto

6 PML Rules: An Alternative Approach for CAT XoL Premium

1. Introduction

- To provide basic information about pricing of reinsurance
- The main approach is to always consider
  a) frequency of losses, and
  b) severity of losses

Types of Reinsurance

$$S = \sum_{i=1}^{I} R_i = \sum_{n=1}^{N} X_n = \sum_{k=1}^{N^*} X_k^*$$

- 1.1 Quota Share, Variable Q/S

$$S = C * S, \quad S' = (1-C) * S \ (0 < C < 1)$$

1.2 Surplus

$$S = \sum_{i=1}^{I} C_i R_i \,, \quad S' = \sum_{i=1}^{I} \min(1 - C_i,\ m v_0 / v_i) R_i$$

Types of Reinsurance

- 1.3 Excess of Loss(XoL)

$$S = \sum_{n=1}^{N} \min(X_n\,,\; a)$$

a, s₀ = deductible

- 1.4 Cumulative XoL

$$S = \sum_{k=1}^{N^*} \min(X_k^*\,,\; a^*)$$

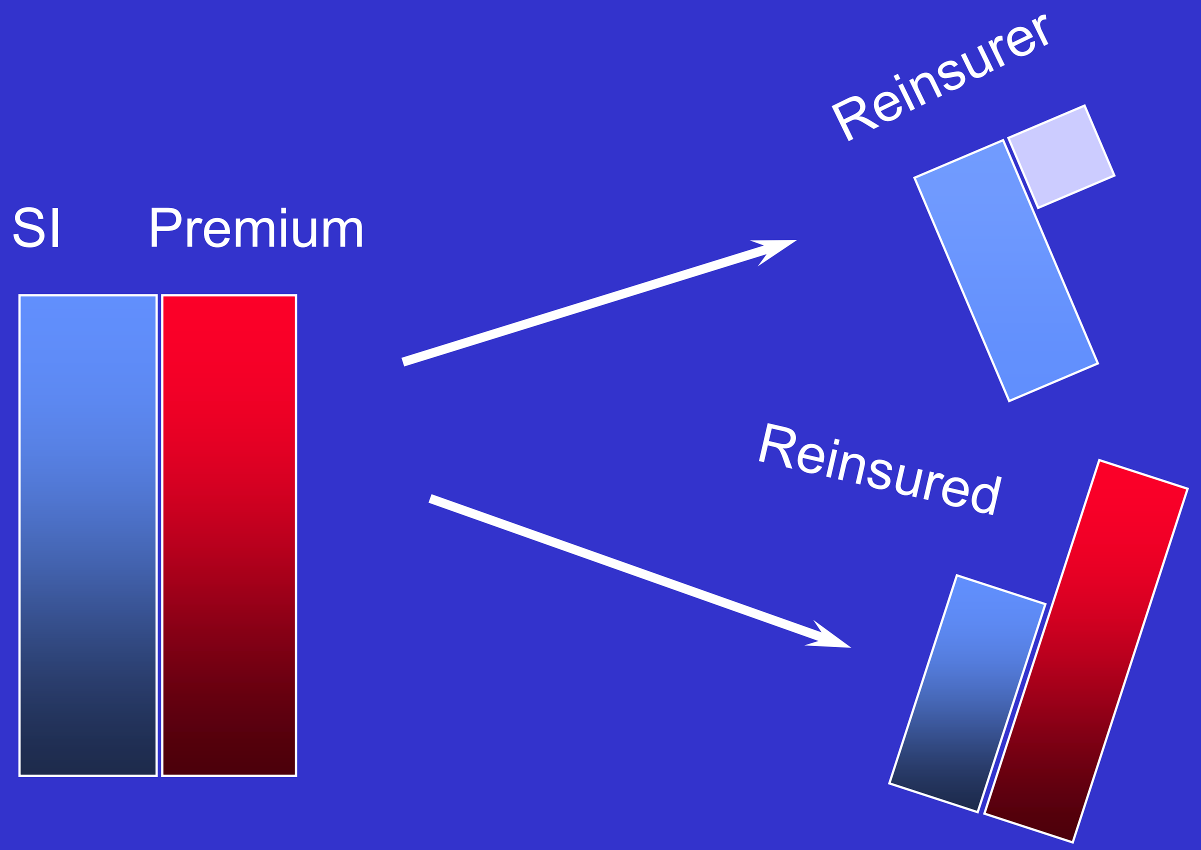
- 1.5 Stop Loss

$$S = \min(\,S\,,\; s_0\,)$$

2. Exposure Rating Method

- Exposure Rating: can be used for Excess-of-Loss treaties with no loss experience or with experience that is statistically not meaningful, should be used in conjunction with Burning Cost and Pareto methods.
- The risk premium paid to the direct insurer is divided into two parts according to the distribution of the exposure to direct insurer and reinsurer
- Rating Scale (Exposure Curve) describes the distribution of the exposure

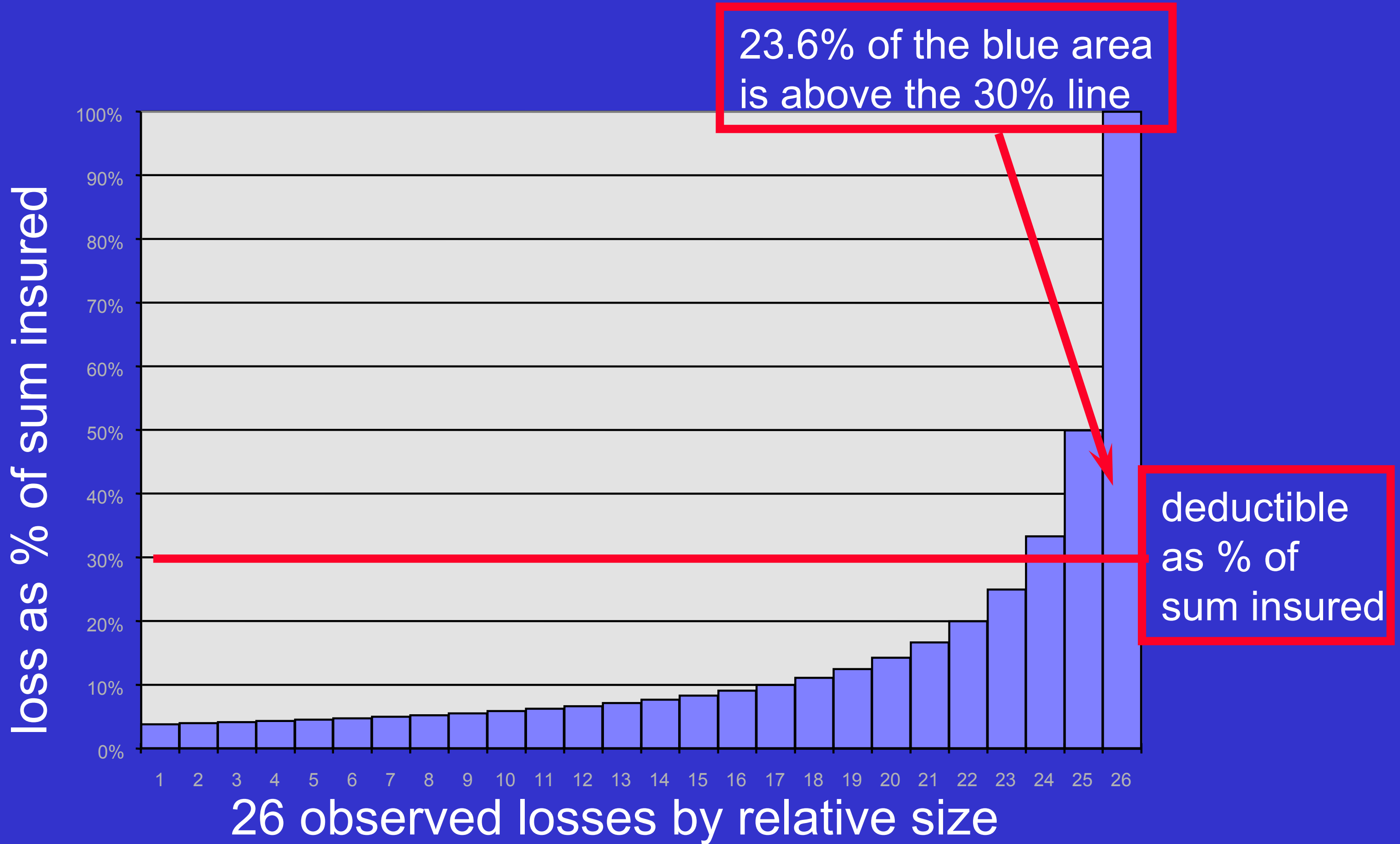
Premium Apportionment



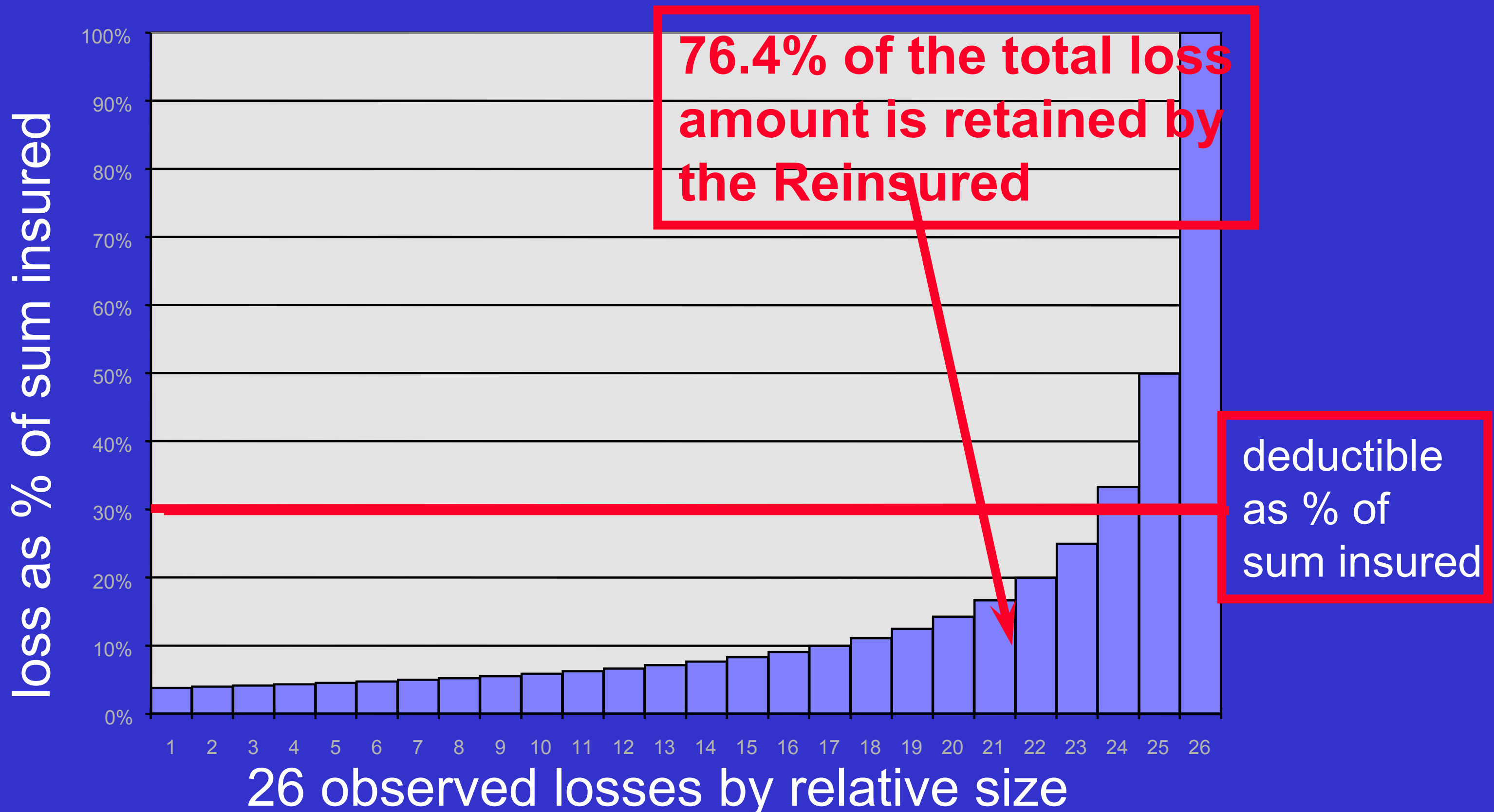
Rating Exercises: Exposure Rating

- How to determine an exposure curve?
  - Simplified Example:
    Let´s consider a portfolio assuming that a 100% loss occurs with exactly the same frequency as any other loss (see next slide)
  - How is the split of the exposure at a priority of 30% of the sum insured?

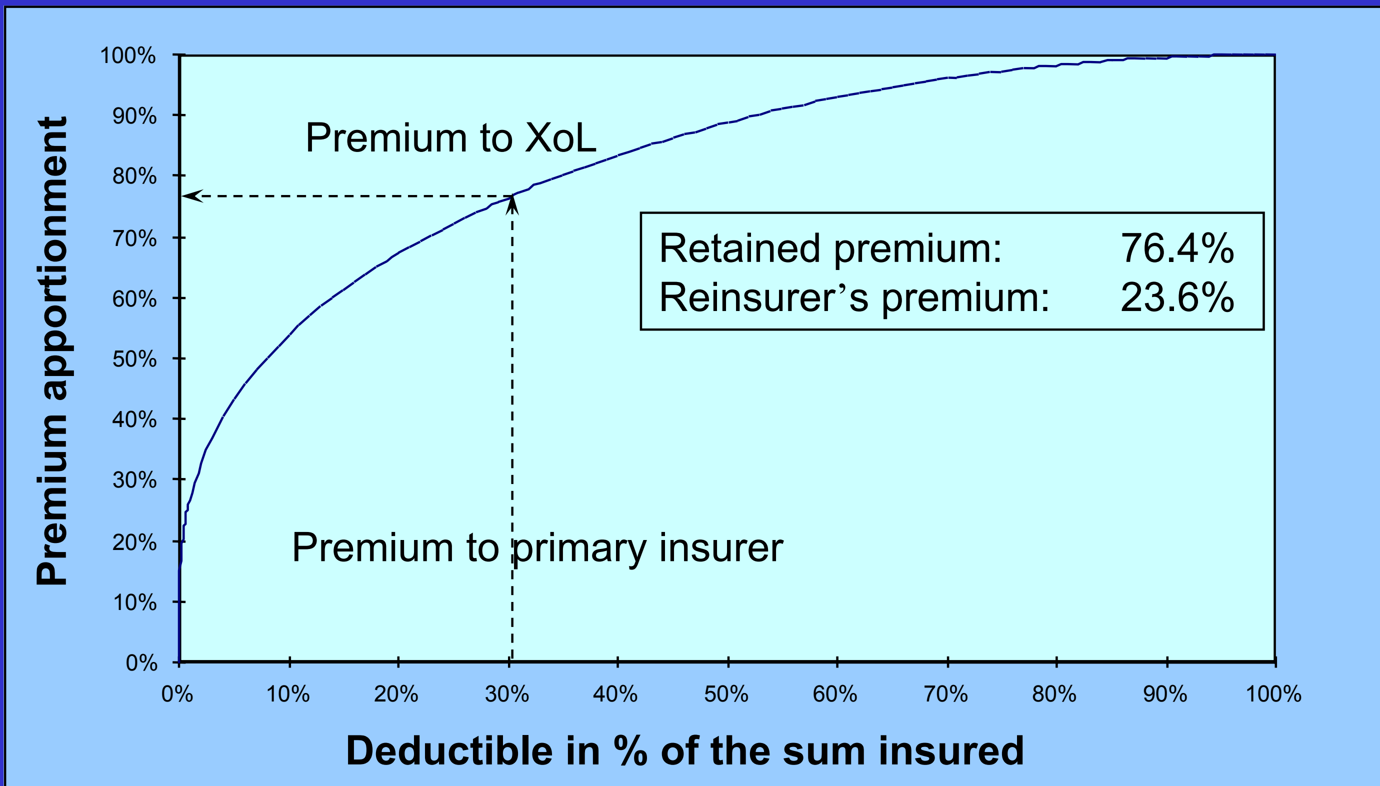
Determining an Exposure Curve



Determining an Exposure Curve



Determining an Exposure Curve



Determining an Exposure Curve

Summary:

Priority as % of SI	Insurer´s Share	Reinsurer´s Share
0.00%	0.00%	100.00%
25.00%	72.15%	27.85%
33.33%	78.75%	21.25%
50.00%	88.79%	11.21%
66.67%	95.20%	4.80%
75.00%	97.25%	2.75%
100.00%	100.00%	0.00%

Example

Determine a rate for the layer $500.000 XS $500.000 for the following portfolio:

Sum Insured	Number of Risks	Risk Premium
$500,000	2,000	$300,000
$750,000	1,200	$200,000
$1,000,000	800	$150,000

Example

Using our previously developed Exposure Curve leads us to the following overview:

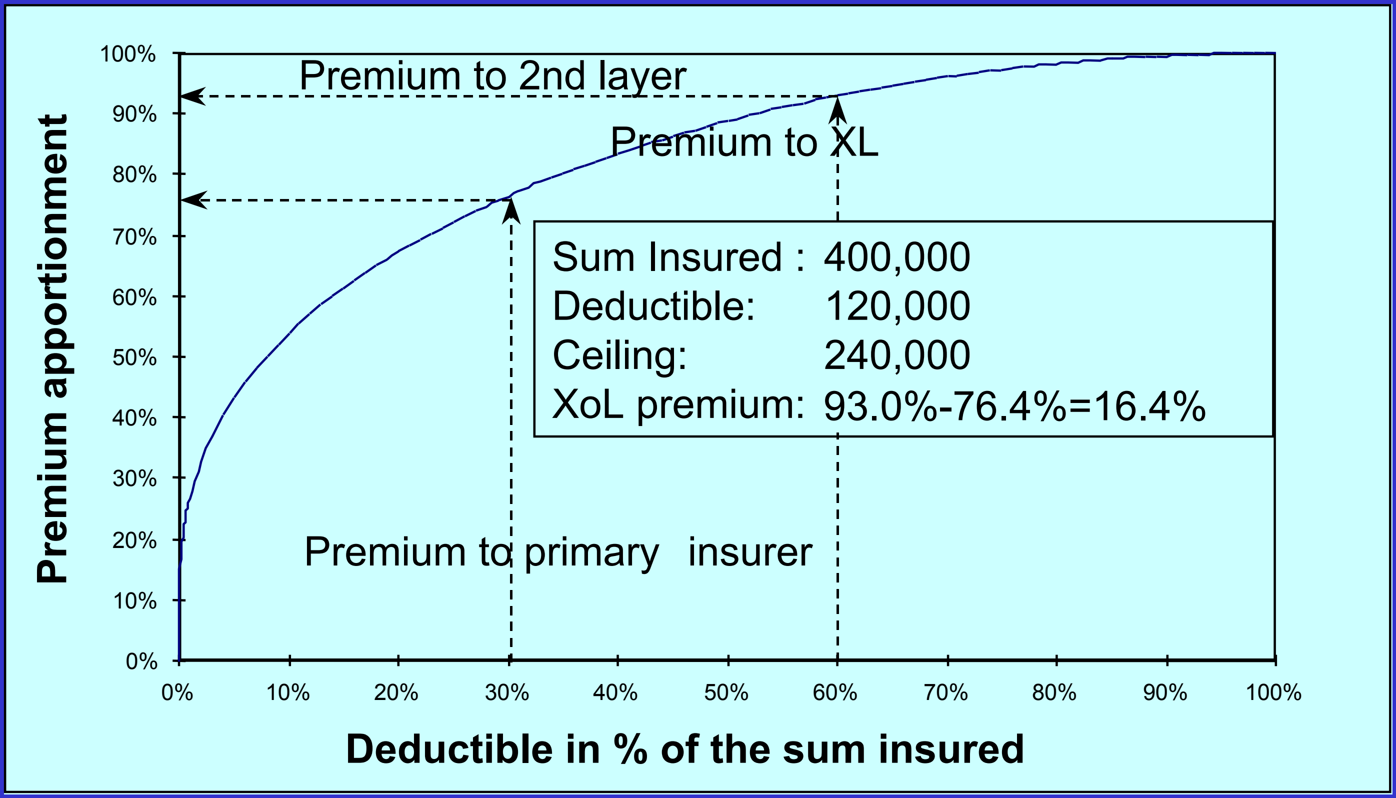
Original Sum Insured	Retention as % of SI	Reinsurer´s Share	XoL Risk Premium
$500,000	100.00%	0.00%	$0
$750,000	66.67%	4.80%	$9,600
$1,000,000	50.00%	11.21%	$16,815
		Total:	**$26,415**

Result: $26,415 / $650,000 = 4.06% (Risk Rate)

Exposure Rating (Single Risk)

- How to rate an XoL layer?
  - calculate deductible and ceiling (= deductible + limit) of the layer as % of the risk's sum insured
  - look up these two exposure figures in the curve/table and find the corresponding % of the premium
  - take the difference of the two premium % figures and apply it to the original premium of the risk
  - this is the premium for the XoL Reinsurer

Example



Premium to 2nd layer
Premium to XL
Sum Insured : 400,000
Deductible: 120,000
Ceiling: 240,000
XoL premium: 93.0%-76.4%=16.4%
Premium to primary insurer
Premium apportionment
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
0%
10%
20%
30%
40%
50%
60%
70%
80%
90%
100%
Deductible in % of the sum insured

Rating Exercises : Exposure Rating

- Of course, our assumption that all loss events (as defined in relation to sum insured) have the same probability of occurrence does not hold true in reality.
- In many lines partial losses occur with far greater frequency than total losses.
- To obtain an accurate rating, it is necessary to know or determine the distribution of loss probabilities for the risks or portfolio covered - this represents the true difficulty of this method.

Rating a Portfolio

- Ideally, the exposure rating procedure should be applied to each individual risk of a portfolio
- For the sake of simplicity, the entire portfolio is condensed into a "risk profile"
- Risks are grouped into bands according to their sum insured; only the number of risks, aggregate premium and aggregate sum insured is shown

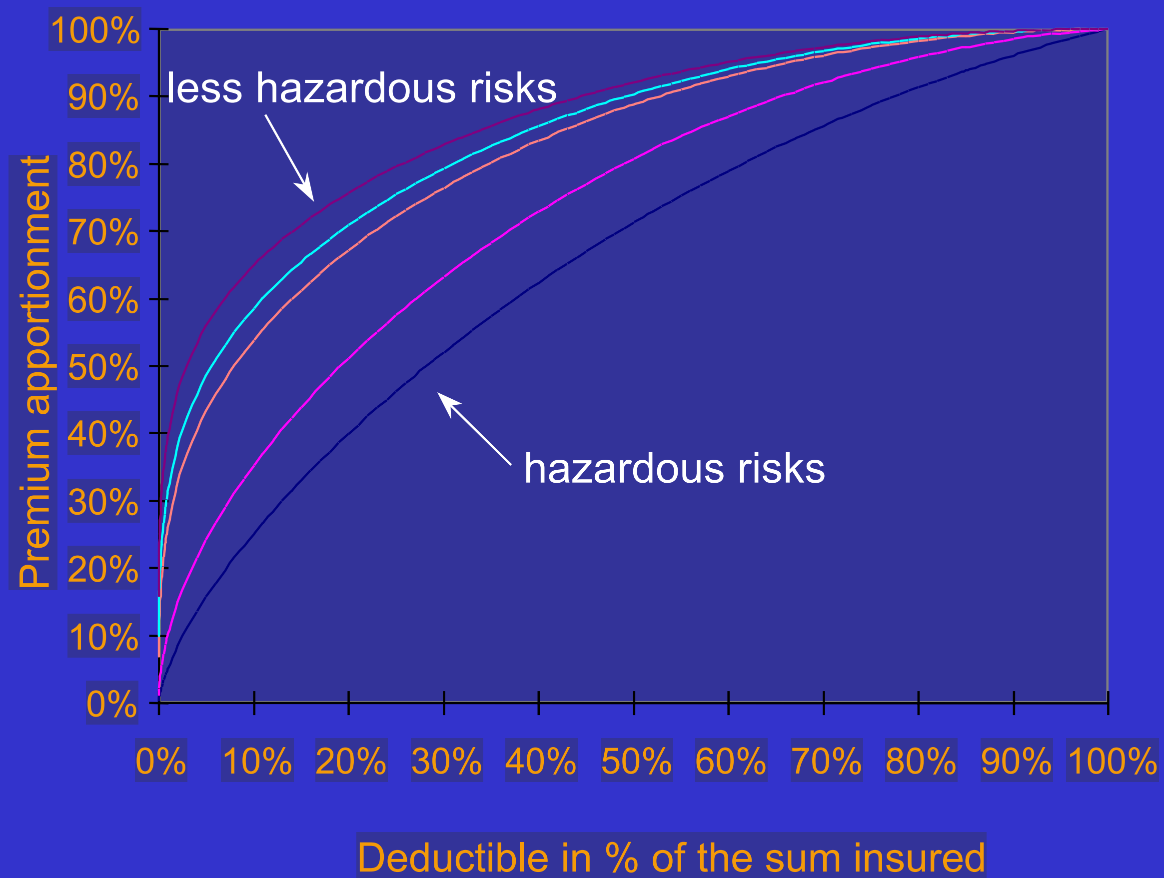
Risk Profile

Sums Insured (000)	No. of Risks	Premium
$1,000	822	$754,185
$2,000	345	$994,118
$3,000	292	$1,286,260
$4,000	167	$830,575
$5,000	101	$638,573
$6,000	89	$599,638
$8,000	92	$734,804
$10,000	35	$318,780
$15,000	7	$75,863
$20,000	3	$57,855
Total	1,953	$6,290,651

Exposure Quotation - Problems

- relevance of the risk profile
  - inflation
  - portfolio changes
- Sum Insured / PML base
- known accumulations (Fire/Loss or Profits)
- taking into account proportional reinsurance
- sufficiency of original premium
- applicability of the curves
  - origin
  - high / low sums insured
  - hazardous / less hazardous risks

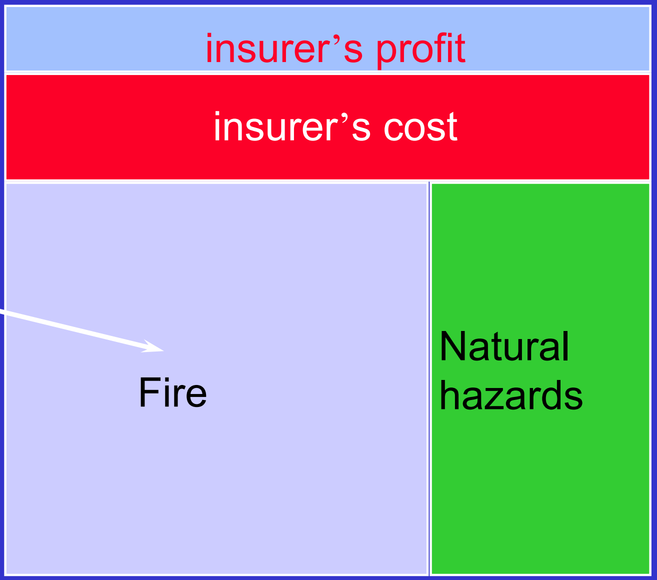
Exposure Quotation - Problems



Different Exposure Curves

Exposure Quotation - Problems

Risk premium
basis for fire
XoL treaty



Sufficient Risk Premium?

3. Burning Cost Method

Experience Rating

- Used for treaties with a credible loss history, should be used in conjunction with Exposure and Pareto Method, if possible.
- Assumes that future losses will be similar to historical loss experience (or can at least be modeled by trend factors).
- Definition:
  Burning Cost is the sum of all losses to an XoL treaty as a percentage of the corresponding subject premium.

Rating Exercises : Burning Cost Method

- Informationed required
  - Loss information for a <u>minimum period of five years.</u> However, the longer the period obtained, the better.
  - The Basic Premium
  - The individual loss information from ground up
  - The Consumer Price or Wage Index

Example

- Layer to be rated $1,000,000 xs $500,000 any one risk, est. Subject Premium 2004 $31,000,000:

Subject Premium Income		Claims in excess of $350,000	
1999	$23,654,158	1999	$465,000
2000	$24,745,797	1999	$1,250,000
2001	$25,887,816	2001	$485,000
2002	$27,082,539	2002	$1,485,000
2003	$28,332,398	2002	$756,000
		2003	$465,000

Example: $1,000,000 xs $500,000

Revaluation Index			Revalued Subject Prem. Income		Revalued Claims	Revalued Claims to Layer
1999	100.00					
2000	103.50	1999	$28,093,720	1999	$552,274	$52,274
2001	107.12	2000	$28,396,372	1999	$1,484,608	$984,608
2002	110.87	2001	$28,702,284	2001	$537,728	$37,728
2003	114.75	2002	$29,011,492	2002	$1,590,769	$1,000,000
2004	118.77	2003	$29,324,032	2002	$809,846	$309,846
				2003	$481,275	$0

Example: $1,000,000 xs $500,000

	Revalued Subject Prem. Income	Total Revalued Claims to Layer per year	% of SPI
1999	$28,093,720	$1,036,882	3.69%
2000	$28,396,372	$0	0.00%
2001	$28,702,284	$37,728	0.13%
2002	$29,011,492	$1,309,846	4.51%
2003	$29,324,032	$0	0.00%
Total	$143,527,900	$2,384,456	1.66%

Burning Cost: Size of the portfolio

- **This method takes into consideration portfolio growth**

Subject Premium Income 2003: $10,000,000
Number of risks in 2003: 100,000

Number of risk losses -> $250,000 in 2003: 5

est. Subject Premium Income 2004: $18,000,000
est. Number of risks in 2004: 160,000

Number of risk losses expected -> $250,000 in 2004: 5.6

Disadvantages of Burning Cost

- Unused cover:
  - Pareto or Exposure Methods.
- Incurred But Not Reported losses
- Changes in reinsurance program
- Changes in reinsured portfolio
- Changes in original rates
- Choice of observation period

Burning Cost - Example

- With historical premium information we would now be able to calculate the average Burning Cost Rate.
- Again this rate would have to be loaded for costs, profit, etc.

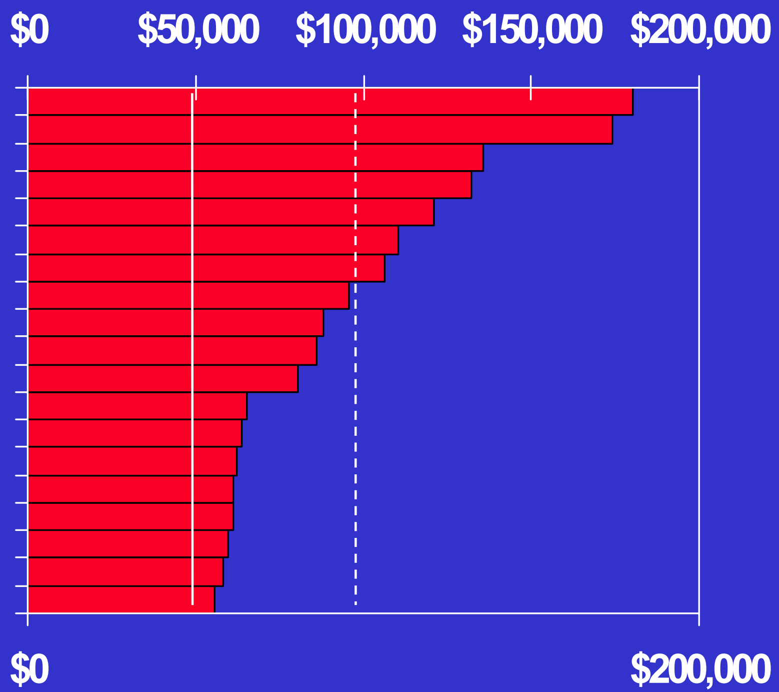
4. Pareto Method: Distribution Approach

- **Idea: extrapolation of observed (or modeled) losses using a Pareto curve. These “modeled” losses are also known as “as-if” losses (losses that have not occurred but could occur according to the curve)**
- **Should be used in conjunction with Exposure & Burning Cost method, if possible.**

The Pareto Distribution Model

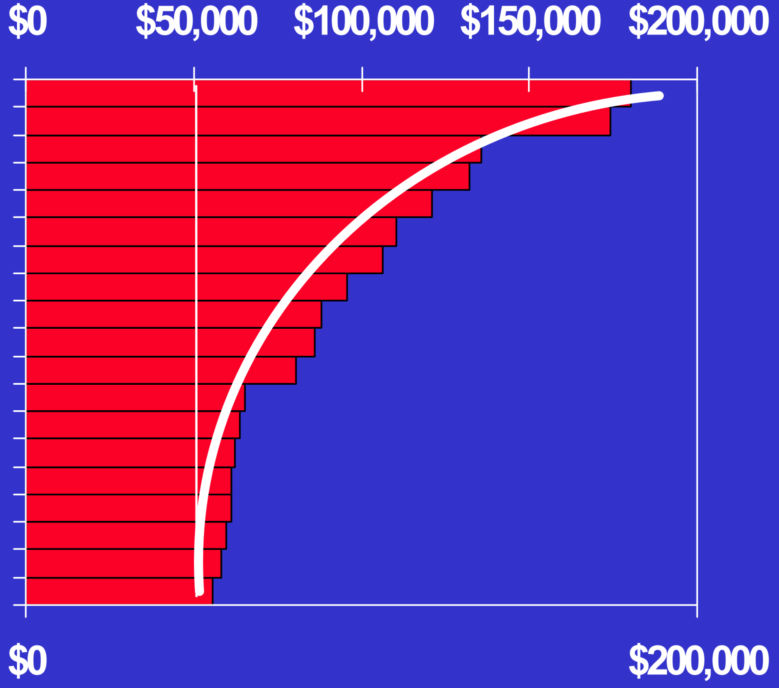
- **as in the Burning Cost method, past claims are re-calculated on an 'as if' basis (e.g. inflation, portfolio growth, priority reinsurance)**
- **an observation point 'k' is defined in a way that all 'as if' claims exceeding 'k' are known**
- **in order to visualize the model, all 'as if' claims are ordered by size and stacked in a bar diagram (see the following picture), resulting in a claims size distribution**

Claims Size Distribution



- 19 claims are in excess of the observation point k=$50,000
- they form a step function
- the probability for a claim excess of $50,000 to exceed $100,000, is 7/19 or 37 %.

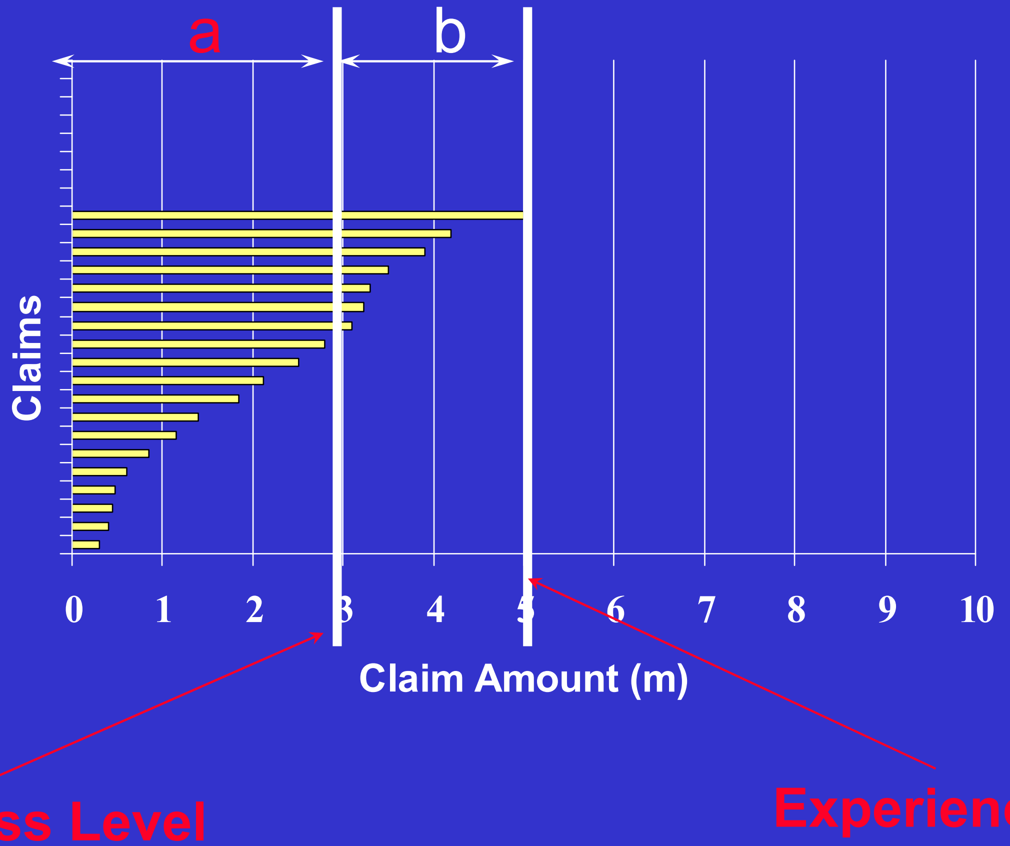
Claims Size Distribution



- the step function is approximated by an unlimited Pareto curve
- using the Pareto curve, the probability for claims in excess of the largest observed claim can be calculated

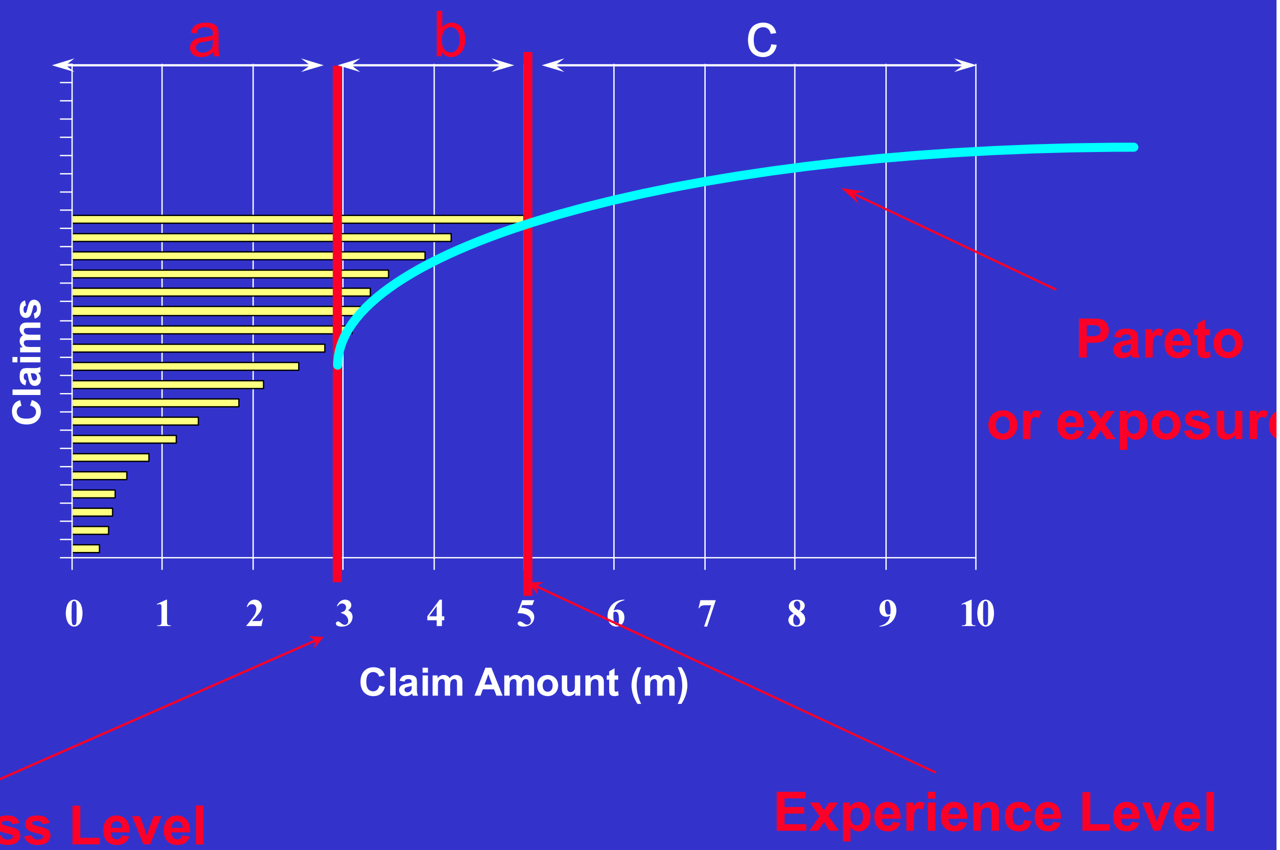
Pricing the Treaty

- a = basic loss ratio
- b = burning cost



Pricing the Treaty

- a = basic loss ratio
- b = burning cost
- c: = unused capacity



Mathematical Formula

The (conditional) probability that a loss "X" exceeding the observation point "k" is also in excess of an amount x equals

$$1\text{-}F(X) = 1 - \left(\frac{k}{k+x}\right)^{\alpha}$$

Advantages of the Pareto Method

- As opposed to the burning cost method, the Pareto method considers claims below the deductible and the exact amount of claims exceeding the ceiling
- By extrapolating the curve, layers or parts of layers without loss experience can be priced
- The distribution is defined by 2 parameters (k, α)
  - layers can be compared
  - alternatives can be calculated consistently
  - sometimes it is possible to find market parameters (e.g. Cat)

Disadvantages of the Pareto Method

- The actual claims experience is smoothed
- Information is lost because of the reduction to 3 parameters
- Sometimes, the Pareto curve is a poor approximation to the observed claims
- Results depend on the choice of the observation point
- A price is calculated even for non-exposed areas (e.g. above the retention)

5. Alternatives of Pareto

- Generalized Pareto
- Burr distribution $F(x)=1-\left(1-\left(\frac{x}{x_0}\right)^{\tau}\right)^{-\alpha} \quad for\ x>0, x_0>0, \alpha>0, \tau>0$
- Compound Poisson
- Extreme Value Theory(EVT): Gumbel Dis.

6. PML Rule: an Alternative approach for CAT XoL Premium

$$1 - F\left[\frac{A + \alpha\rho_0 - R_{premium} + P_{payoff}}{\alpha}\right] = p^*$$

$$1 - F\left[\frac{A + \alpha\rho_0 - (1 + l_R)\int_{L_0(\alpha)}^{L_1(\alpha)}\left[1 - F\left(\frac{L}{\alpha}\right)dL + \Delta\right.}{\alpha}\right] = p^*$$

A= $initial\ reserves,\ \ \rho_0 = premium = (1 + l_I) \times\ Expected\ \ losses$

$R_{premium} = reinsurance\ \ premium = (1 + l_R)\int_{L_0}^{L_1}[1 - F(L/\alpha)]dL$

$L_1 = L_1(\alpha) = L_0(\alpha) + \Delta\ ,\ \ R_{payoff} = reinsurance\ \ payoff$

$p^* = target\ ruin\ probability,$ $\boldsymbol{\alpha = ratio\ of\ TSI}$