Least Squares Monte Carlo (LSMC) life and annuity application

Prepared for
Institute of Actuaries of Japan

February 3, 2015

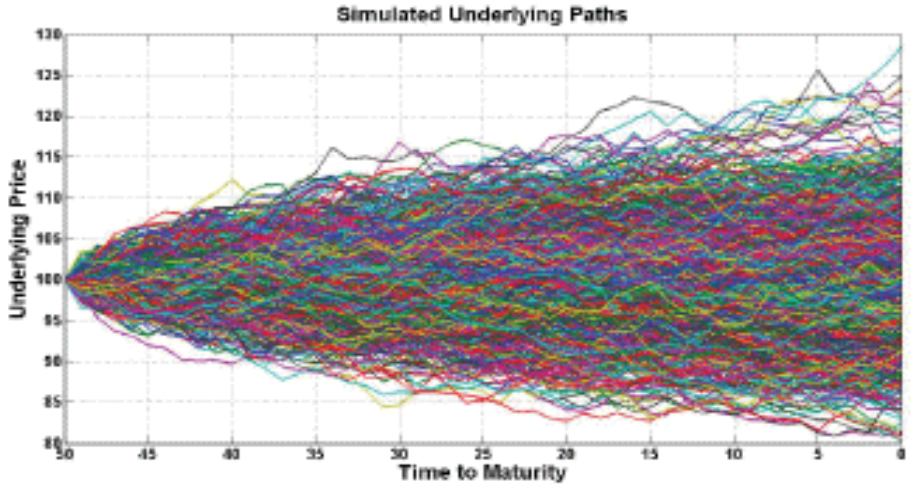
Milliman

Agenda

- A bit of theory
- Overview of application
- Case studies
- Final remarks

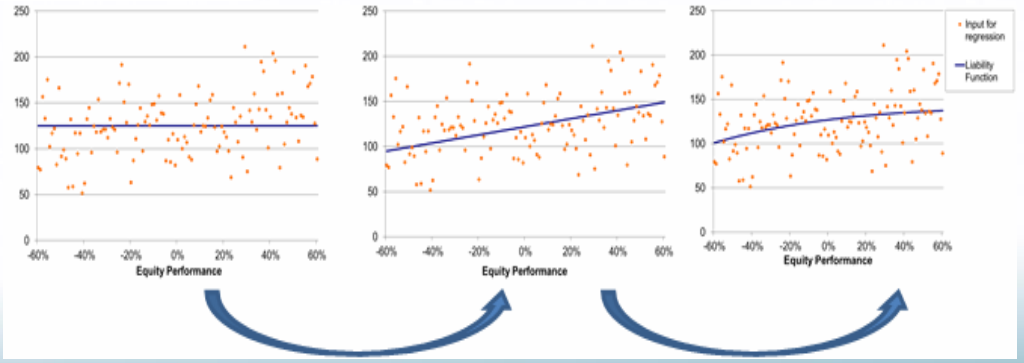
Least Squares Monte Carlo Theory

- In life and annuity application, LSMC derives a closed form solution for any stochastic calculation as a function of its risk drivers.



Monte Carlo Simulation

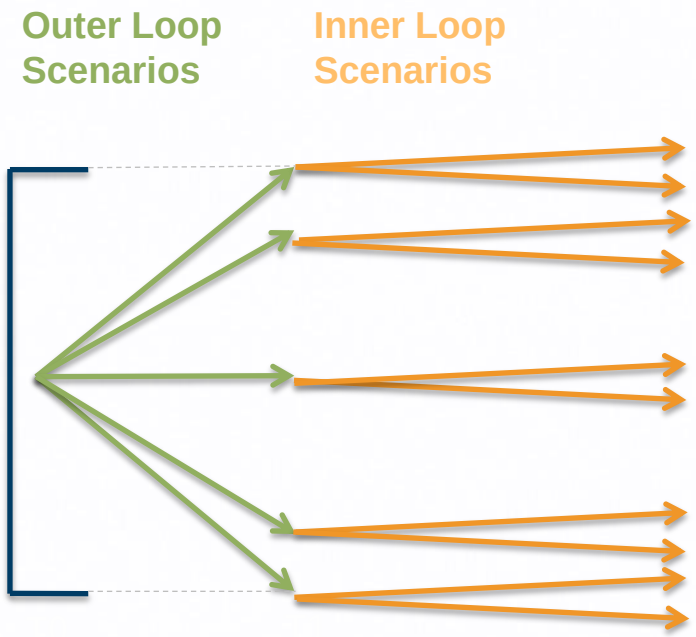
Least Squares Regression



Milliman

Least Squares Monte Carlo Theory

- Use many (e.g. 10,000) outer scenarios with few (e.g. 10) inner scenarios
  - Outer scenarios are shocks to current market/non-market condition
  - Inner scenarios are economic scenarios
- Resulting liability values are not reliable on their own due to the low number of inner scenarios.
- Collect a large amount of inaccurate information

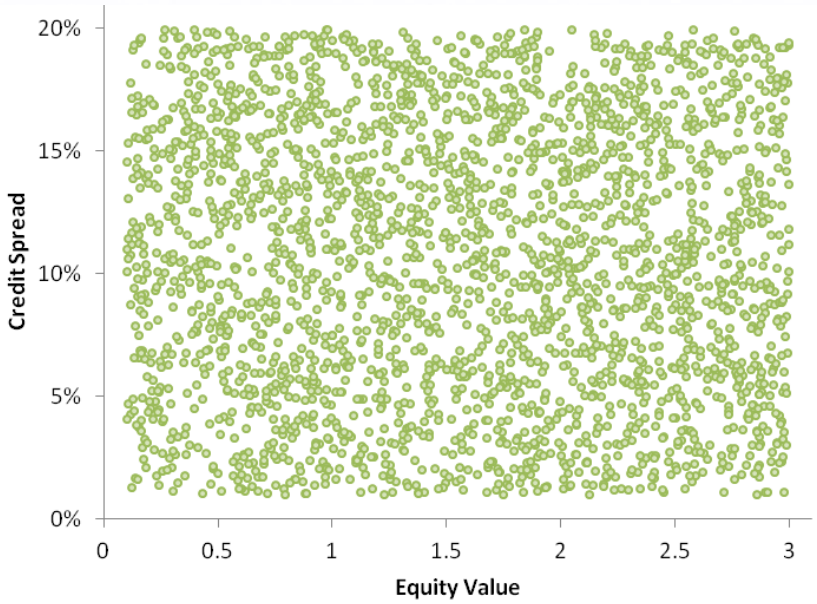


Milliman

Least Squares Monte Carlo Theory

- Position of outer scenario shocks are uniformly distributed over space of possible risk driver values.

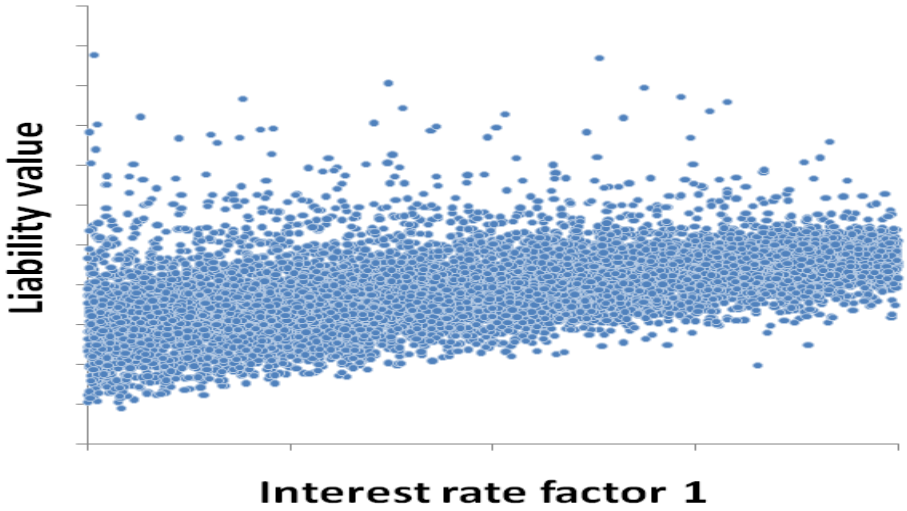
- No need for an economic view on joint positions of risk drivers and their probabilities
- Information is evenly spread over space of risk drivers
- Leads to robust estimates



Milliman

Least Squares Monte Carlo Theory

As a result, we get many inaccurate liability values



- Need to remove sampling error and uncover relationships
- Idea is to "smooth through" inaccurate fitting values
  → Use least squares regression
  → Liability function is linear combination of polynomial terms and cross terms

Milliman

Least Squares Monte Carlo Theory

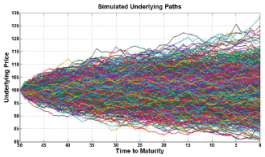
Least Squares regression allows for *automated* model selection:

- Start with trivial polynomial
- Successively add further terms to polynomial
- Decision when to stop this process and which term to choose is purely based on statistical criteria

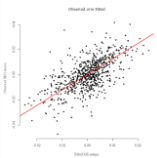
Random error in single liability values is cancelled out (in mean) and true relationship between risk drivers and liability values is unveiled.

Milliman

Least Squares Monte Carlo Theory
Process outline



Identify risk drivers → **Assess shock ranges** → **Generate scenarios** → **Monte Carlo actuarial calculation**



$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots\ldots + a_1 x^1 + a_0 x^0$$

LSMC → **Validate results** → **Report**

Milliman

Least Squares Monte Carlo Theory

Three layers of validation:

1. Compare function values with Brute-Force (full MC) values
   - Hard benchmark, computationally demanding
2. Judge overall shape of curve
   - Overall level, monotonicity, slope, curvature, interactions
3. Derive confidence intervals of function values
   - How much randomness is left?
   - Detect misfits, outliers, data handling errors,…

Overview of applications

In life and annuity application, LSMC derives a closed form solution for any stochastic calculation as a function of its risk drivers.

- Efficient Modeling
- Avoid frequent Large runs
- Enable stochastic on stochastic projection

Overview of applications

- Real time monitoring (daily or what now scenario)
  - Daily solvency monitoring
  - Profitability tracking
  - Capital position
  - ALM duration mismatch
  - Hedge targets
  - Attribution reporting

- Forecasting (future or sensitivities)
  - Pricing
  - Business forecast
  - ORSA projections

Milliman

Case Studies

Case 1: German Participating Whole Life – SII

Case 2: US Variable Annuity – Capital monitoring

Case 3: US Fixed Annuity – ALM

Case 4: US Variable Annuity - Stochastic on stochastic

Case 1: German Participating Whole Life - SII

Objective

Daily solvency monitoring under SII

Reason for LSMC

Impossible to run daily economic capital on entire block

Type of calculation

Risk neutral (mean) economic capital

Reflect both market and non-market risks

Case 1: German Participating Whole Life - SII

Identify risk drivers

- **Market risks:**
  - Interest rates (level + volatility)
  - Equity (level + volatility)
  - Credit spreads
- **Non-market risks:**
  - Lapse
  - Longevity
  - Mortality

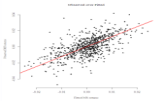
Assess Shock Ranges

Each risk driver is stressed between its upper and lower 99.99966% confidence level under a certain real-world view

Generate Scenarios

50’000 (= 5’000 x 10) fitting scenarios

LSMC



Result of automated calibration process: Polynomial with 94 terms

Milliman

Case 1: German Participating Whole Life - SII

Validate results

1. Compare function values with Brute-Force results

→ Done for 50 different points in risk driver space

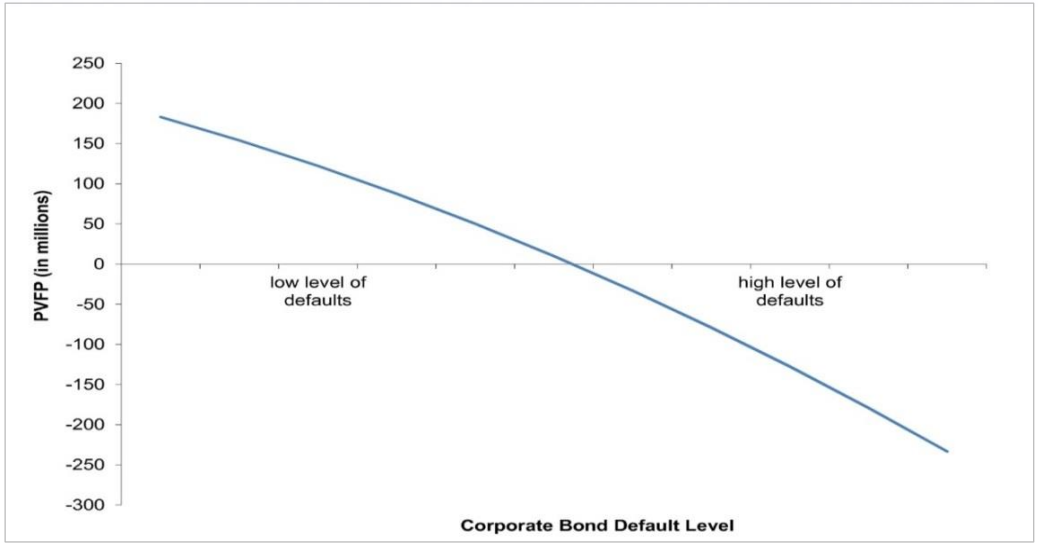
→ Average deviation <2%

Milliman

Case 1: German Participating Whole Life - SII

Validate results

2. Judge overall shape of curve
→ Overall level, slope, curvature, interactions



Present value of shareholder transfers depending on corporate bond default level:

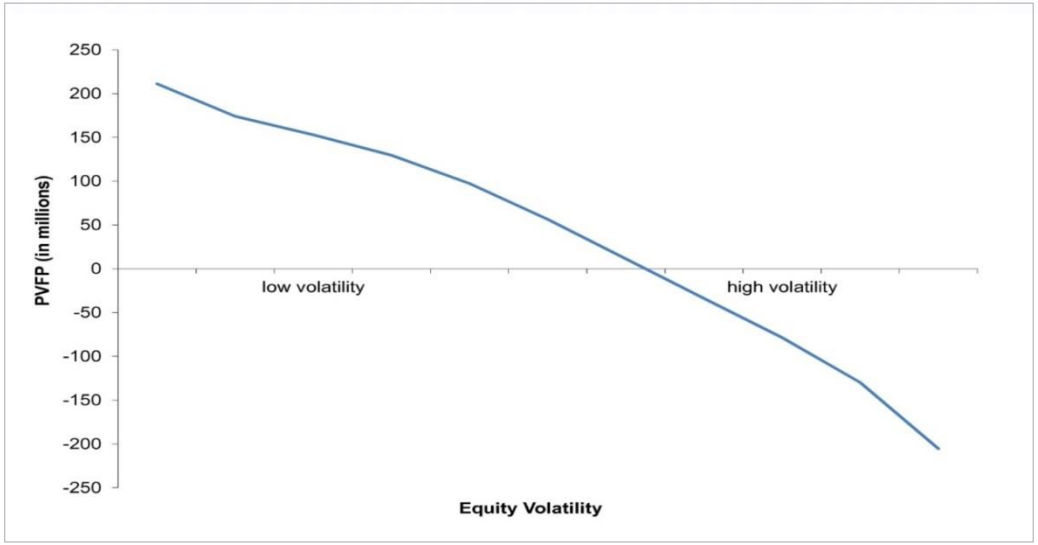
✓More defaults lead to more losses

Milliman

Case 1: German Participating Whole Life - SII

Validate results

2. Judge overall shape of curve
→ Overall level, slope, curvature, interactions



Present value of shareholder transfers depending on equity volatility:

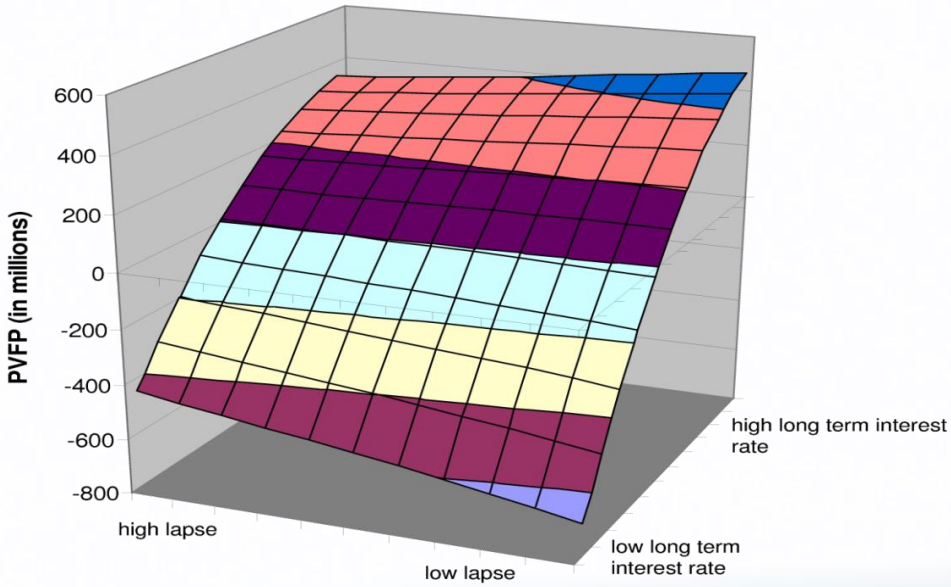
✓Higher volatility leads to more losses

Milliman

Case 1: German Participating Whole Life - SII

Validate results

2. Judge overall shape of curve
→ Overall level, slope, curvature, interactions



Present value of shareholder transfers depending on lapse and interest rate levels:

→Interaction between interest rate level and lapse:

- ✓ **Low interest rate level: Increasing lapse rates help the shareholder to off-load some of the guarantees**
- ✓ **High interest rate level: Increasing lapse rates deprive the shareholder of some capital gains**

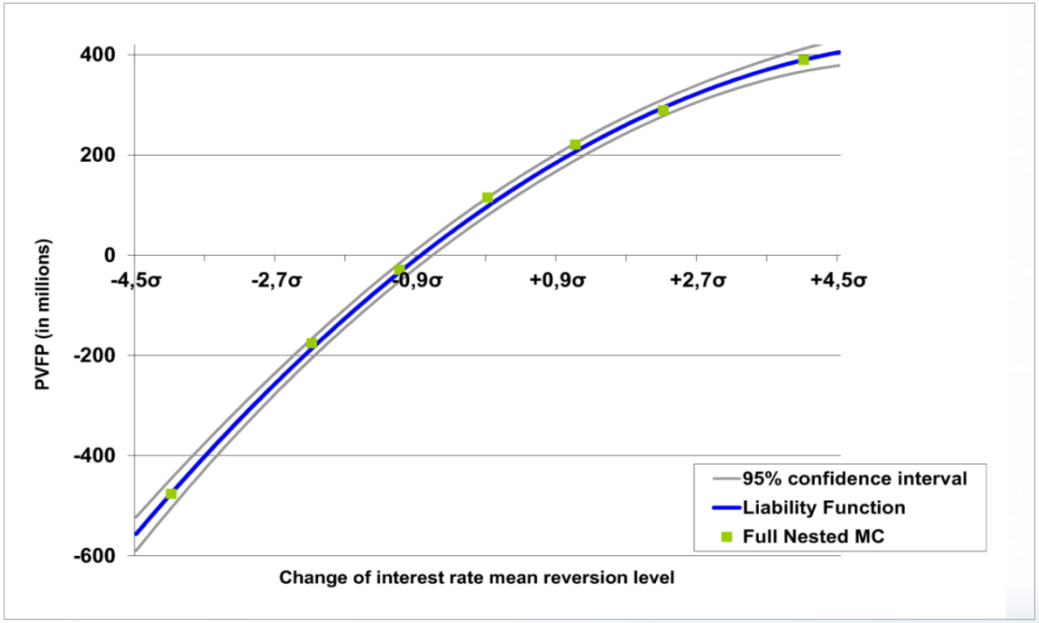
Milliman

Case 1: German Participating Whole Life - SII

Validate results

3. Derive confidence intervals of function values

→ How much randomness is left?



- ✓ Overall shape captured very well
- ✓ Small deviations between true values and LSMC estimates
- ✓ Tight confidence intervals

  →Few randomness left

Milliman

Case 1: German Participating Whole Life - SII

- LSMC works in SII environment; used for daily solvency measurement
- LSMC helps develop intuition
    - Sensitivity to each risk drivers
    - Interaction between risk drivers
- LSMC can help explain attribution of changes

Case 2: US Variable Annuity – Capital Monitoring

Objective

Capital reported quarterly, but wish to monitor monthly;

Wish to understand capital movement when market has big change

Reason for LSMC

No resource (staff or computer) required for monthly or more frequent reporting

Type of calculation

Real world (CTE) economic capital

Reflect only market risks

Case 2: US Variable Annuity – Capital Monitoring

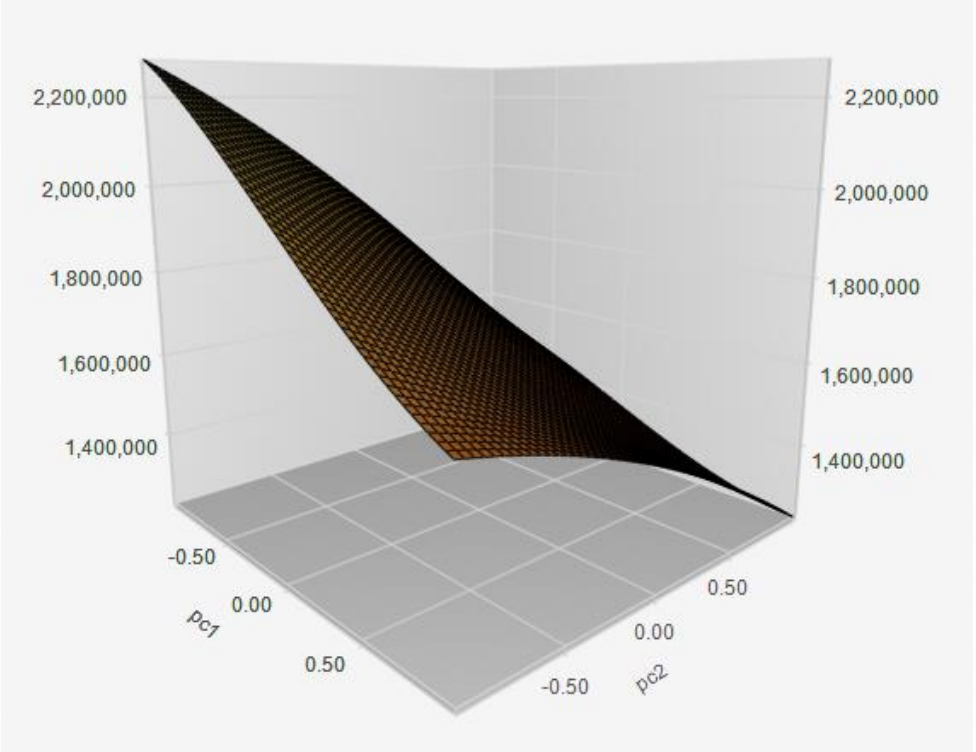
- Setup:
  - VA with deep ITM GLWB (10m Guarantee, 4m account value)
- Risk drivers
  - Domestic Equity
  - International Equity
  - Bond Index
  - Interest Rate (first two principal components)
- Test period
  - 3/31/12 – 9/30/12
  - 8/31/08 – 2/28/09
- Objective
  - Calibrate at beg of each quarter and test how closely LSMC matches actual monthly CTE in the quarter

Case 2: US Variable Annuity – Capital Monitoring

CTE(90) decreases as the yield curve level (PC1) and the yield curve slope (PC2) increase.

Domain of the risk space in graph:

- PC1 +/- 200 bps
- PC2 ranges from full inversion to doubling of the slope.



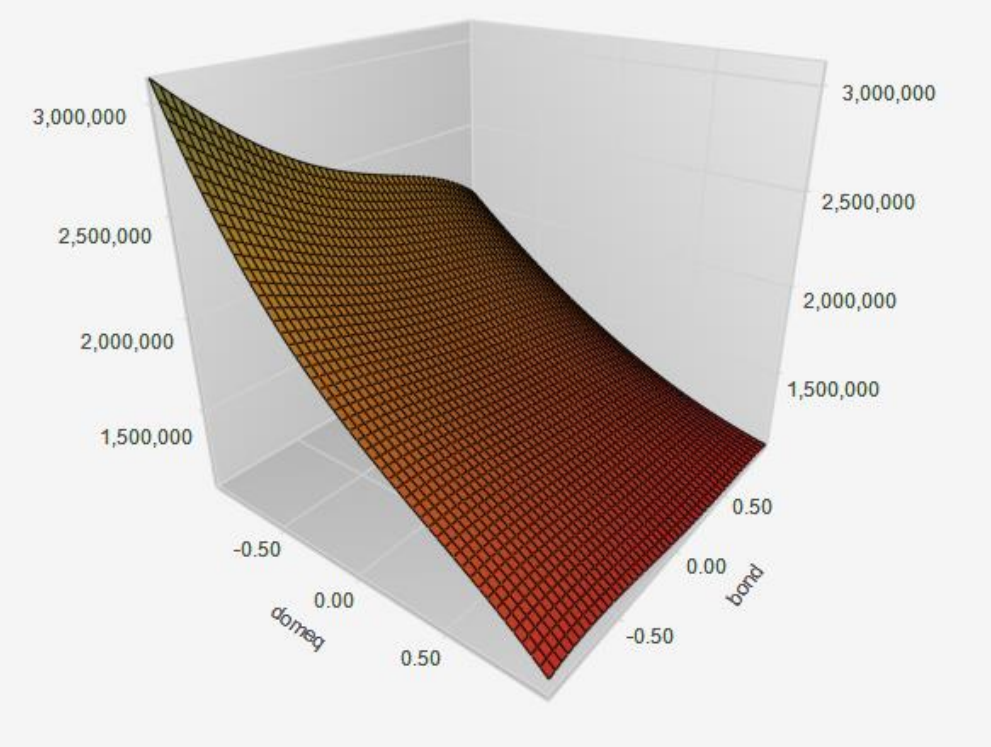
Milliman

Case 2: US Variable Annuity – Capital Monitoring

CTE(90) decreases as equity and bond indices increase.

Domain of the risk space in graph:

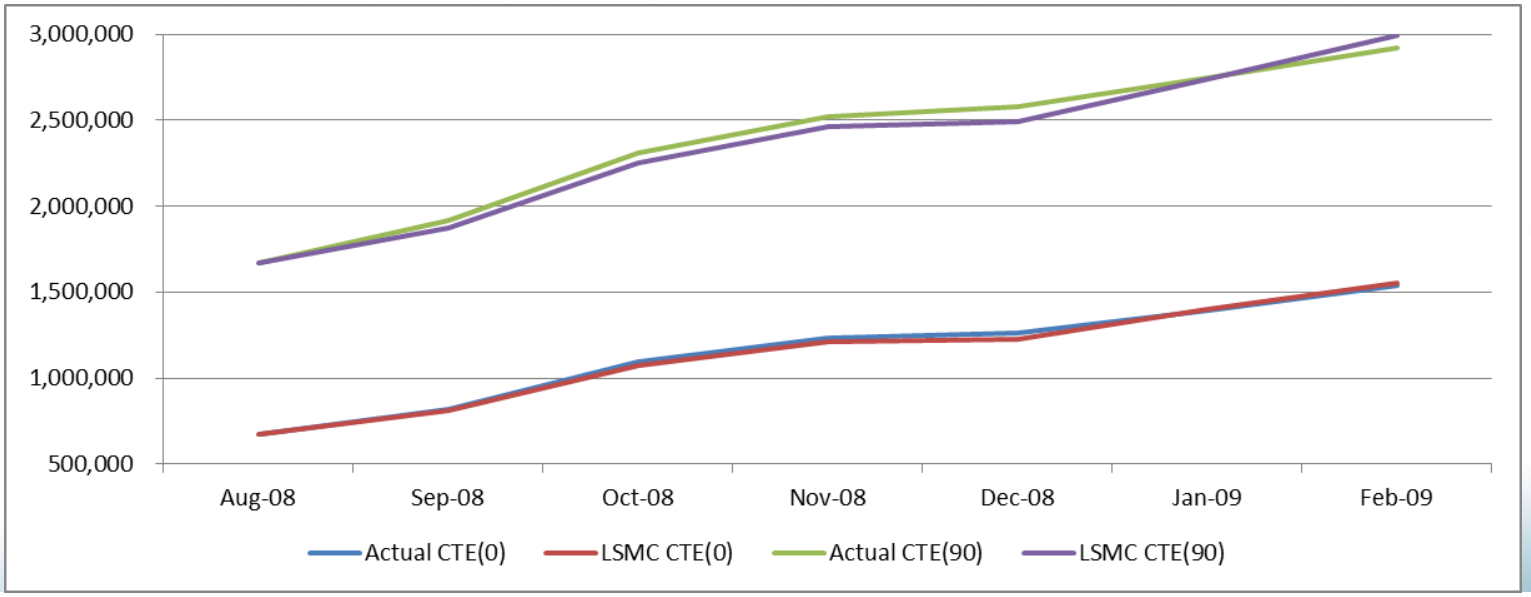
- Domestic equity return: (-60%, +60%)
- Bond fund return: (-60%, +60%)



Milliman

Case 2: US Variable Annuity – Capital Monitoring

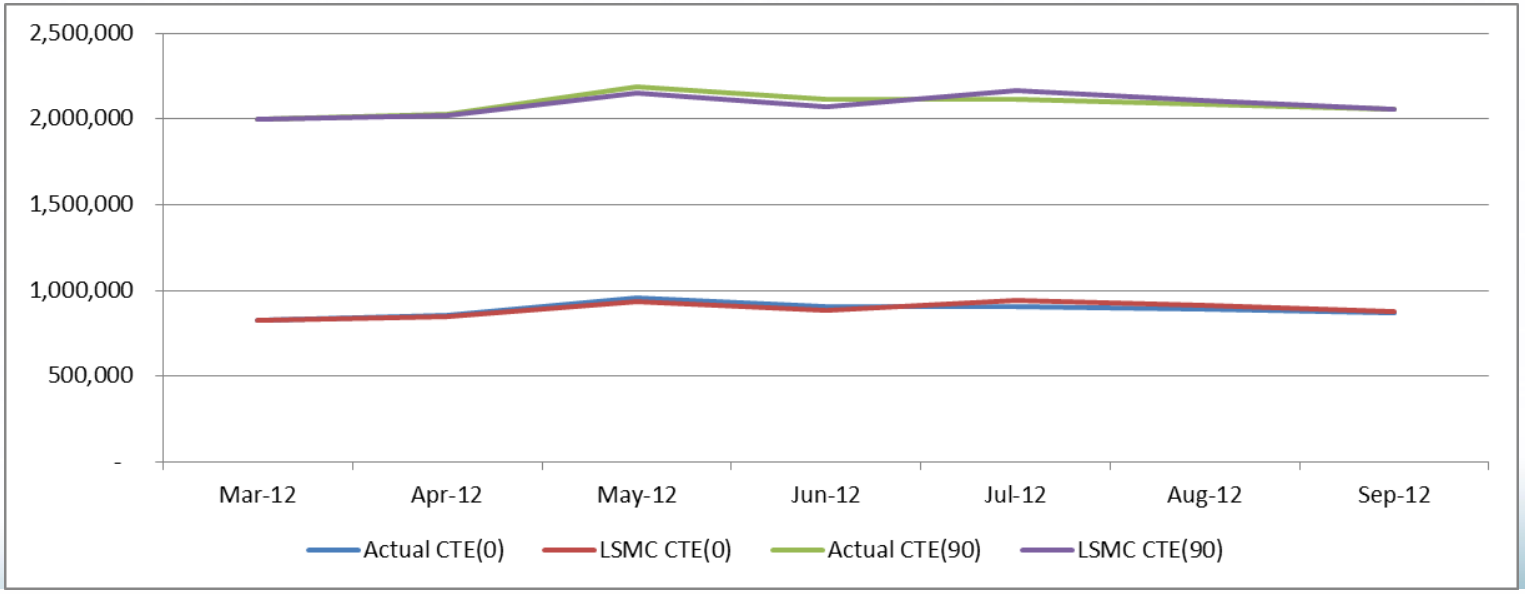
Date	8/31/2008	9/30/2008	10/31/2008	11/30/2008	12/31/2008	1/31/2009	2/28/2009
Short Rate	2.17%	1.78%	1.34%	0.90%	0.37%	0.51%	0.72%
Medium Rate	3.45%	3.38%	3.29%	2.35%	1.87%	2.27%	2.69%
Long Rate	4.43%	4.31%	4.35%	3.45%	2.69%	3.58%	3.71%
Domestic	1,282.83	1,166.36	968.75	896.24	903.25	825.88	735.09
International	55.46	49.12	38.88	36.41	39.63	34.19	30.64
Bond	85.72	84.21	82.29	84.78	90.42	88.63	87.69
Actual CTE(0)	673,037	819,965	1,093,087	1,234,886	1,260,257	1,393,905	1,535,085
LSMC CTE(0)	673,037	809,070	1,070,644	1,213,740	1,225,947	1,398,308	1,555,282
Actual CTE(90)	1,669,260	1,915,224	2,311,306	2,521,270	2,579,166	2,746,363	2,919,130
LSMC CTE(90)	1,669,260	1,876,659	2,249,762	2,459,573	2,493,020	2,740,183	2,994,518



Milliman

Case 2: US Variable Annuity – Capital Monitoring

Date	3/31/2012	4/30/2012	5/31/2012	6/30/2012	7/31/2012	8/31/2012	9/30/2012
Short Rate	0.19%	0.20%	0.18%	0.21%	0.16%	0.16%	0.17%
Medium Rate	1.61%	1.33%	1.03%	1.11%	0.98%	1.01%	1.04%
Long Rate	3.35%	3.12%	2.67%	2.76%	2.56%	2.68%	2.82%
Domestic	1,408.47	1,397.91	1,310.33	1,362.16	1,379.32	1,406.58	1,440.67
International	53.06	51.96	46.17	49.42	49.46	51.04	52.43
Bond	81.60	82.50	83.27	83.32	84.35	84.49	84.64
Actual CTE(0)	824,492	852,740	954,786	907,467	908,110	892,440	870,279
LSMC CTE(0)	824,492	844,369	934,602	883,107	938,851	911,201	880,183
Actual CTE(90)	1,999,785	2,030,838	2,188,788	2,117,304	2,116,912	2,084,463	2,056,270
LSMC CTE(90)	1,999,785	2,019,488	2,154,741	2,073,519	2,164,805	2,110,317	2,060,292



Milliman

Case 2: US Variable Annuity – Capital Monitoring

- LSMC works for real world CTE too
  - More inner loops required
- LSMC helps develop intuition
  - Sensitivity to each risk drivers
  - Interaction between risk drivers
- LSMC can help explain attribution of changes

Real Time Capital management becomes a real possibility

Case 3: US Fixed Annuity – ALM

Objective

Provide ALM manager daily information on asset liability duration mismatch position

Reason for LSMC

No resource (staff or computer) required for daily liability calculation

Type of calculation

Liability duration

Reflect only interest risk

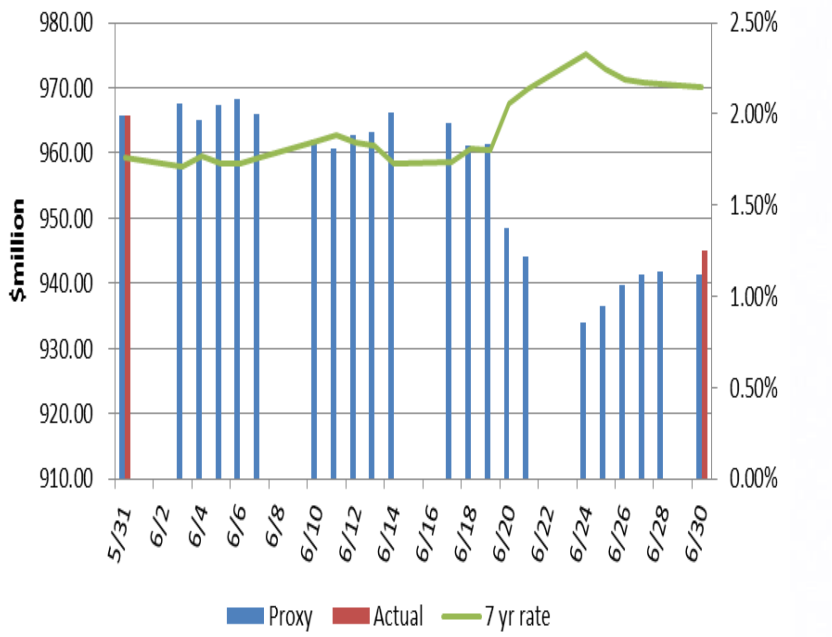
Case 3: US Fixed Annuity – ALM

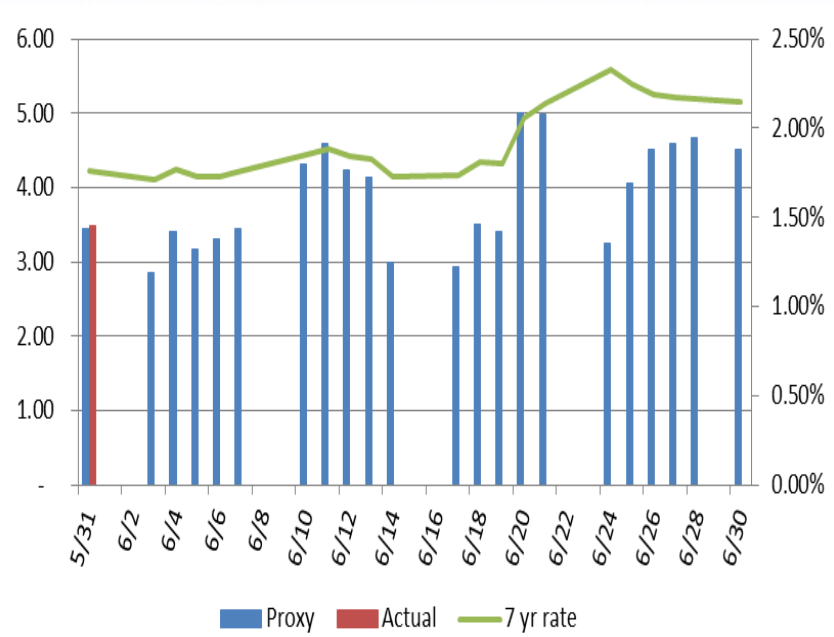
- Setup:
  - $1b fixed annuity
- Risk drivers
  - Key rates (1, 2, 3, 4, 5, 7, 10, 30 year)
- Test period
  - 5/31/13 – 6/30/13
- Objective
  - Calibrate at beg of each month and test how LSMC can be used to monitor duration mismatch position on non-VA

(same concept can be used to track hedging on VA)

Case 3: US Fixed Annuity – ALM

- Validation of PVFB and duration





Milliman

Case 3: US Fixed Annuity – ALM

- DV01 tracking

	DV01								
$thousand	1 yr	2 yr	3 yr	4 yr	5 yr	7 yr	10 yr	30 yr	Duration
5/31/2013	(24)	137	(0)	368	332	(806)	(332)	-	3.45
6/3/2013	(22)	139	1	376	404	(833)	(335)	-	2.86
6/4/2013	(24)	134	0	372	340	(814)	(331)	-	3.41
6/5/2013	(24)	139	1	377	363	(819)	(335)	-	3.16
6/6/2013	(26)	140	0	376	345	(810)	(338)	-	3.31
6/7/2013	(25)	137	(0)	371	331	(806)	(333)	-	3.45
6/10/2013	(26)	127	(6)	355	236	(772)	(322)	-	4.32
6/11/2013	(27)	124	(10)	359	200	(763)	(317)	-	4.59
6/12/2013	(27)	128	(6)	361	245	(777)	(326)	-	4.24
6/13/2013	(27)	130	(5)	364	256	(781)	(327)	-	4.13
6/14/2013	(23)	138	1	381	376	(824)	(333)	-	3.00
6/17/2013	(21)	136	1	380	382	(827)	(329)	-	2.93
6/18/2013	(23)	130	1	382	303	(806)	(319)	-	3.51
6/19/2013	(23)	131	2	385	312	(809)	(320)	-	3.41
6/20/2013	(21)	94	(23)	364	75	(704)	(268)	-	5.00
6/21/2013	(17)	78	(33)	368	28	(672)	(241)	-	4.99
6/24/2013	2	34	(47)	411	(12)	(587)	(155)	-	3.24
6/25/2013	(4)	54	(36)	387	16	(641)	(191)	-	4.07
6/26/2013	(10)	68	(32)	383	27	(669)	(214)	-	4.52
6/27/2013	(11)	72	(29)	366	48	(671)	(226)	-	4.58
6/28/2013	(13)	74	(29)	372	34	(669)	(229)	-	4.67
6/30/2013	(11)	76	(24)	379	70	(700)	(231)	-	4.51

Milliman

Case 3: US Fixed Annuity – ALM

- ALM tracking

$million	Change in:		
Date	Assets	Liabilities	Difference
6/3/2013	1.98	1.70	(0.28)
6/4/2013	(2.77)	(2.50)	0.28
6/5/2013	2.36	2.26	(0.09)
6/6/2013	1.03	1.08	0.05
6/7/2013	(2.39)	(2.34)	0.04
6/10/2013	(3.86)	(4.01)	(0.16)
6/11/2013	(1.24)	(1.44)	(0.20)
6/12/2013	1.91	2.15	0.25
6/13/2013	0.33	0.38	0.05
6/14/2013	3.11	3.00	(0.11)
6/17/2013	(1.61)	(1.63)	(0.02)
6/18/2013	(3.62)	(3.37)	0.25
6/19/2013	0.31	0.29	(0.02)
6/20/2013	(11.87)	(13.19)	(1.32)
6/21/2013	(3.84)	(4.55)	(0.71)
6/24/2013	(10.37)	(10.35)	0.02
6/25/2013	2.16	2.76	0.60
6/26/2013	3.13	3.07	(0.06)
6/27/2013	1.50	1.67	0.17
6/28/2013	0.67	0.50	(0.17)
6/30/2013	(1.12)	(0.45)	0.67
Total	**(24.19)**	**(24.96)**	**(0.77)**

Case 3: US Fixed Annuity – ALM

- LSMC works for different product lines
- LSMC provides an alternative to daily ALM or hedging

Case 4: US Variable Annuity – Stochastic on Stochastic

Objective

Project reserve and required capital in a stochastic exercise

e.g., pricing, business forecast, capital forecast (ORSA), etc

Reason for LSMC

Stochastic on stochastic very computationally intensive

Type of calculation

Stochastic on stochastic

Reflect market risks and aging of portfolio

Case 4: US Variable Annuity – Stochastic on Stochastic

- Variable Annuity (VA) with Guaranteed Lifetime Withdrawal Benefit (GLWB)
- Risk drivers captured in the calibration:
  - Domestic equity index
  - International equity index
  - Bond index
  - 1 Yr. Swap rate
  - 10 Yr. Swap rate
  - Lapsed account value (AV) since issue
  - Total AV inforce
  - Lapsed lives since issue
  - Total lives inforce
  - Total net amount at risk (NAR)
  - Total net amount out of risk (NAOR)

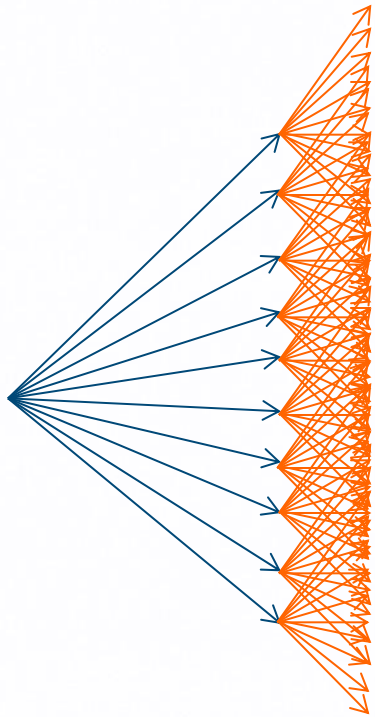
NAR = Max[PV(GLWB) – AV,0]; NAOR = Max[AV – PV(GLWB),0];
PV(GLWB) = present value of future withdrawals at modeled mortality and discounted at the 10 Yr. Swap rate

Case 4: US Variable Annuity – Stochastic on Stochastic

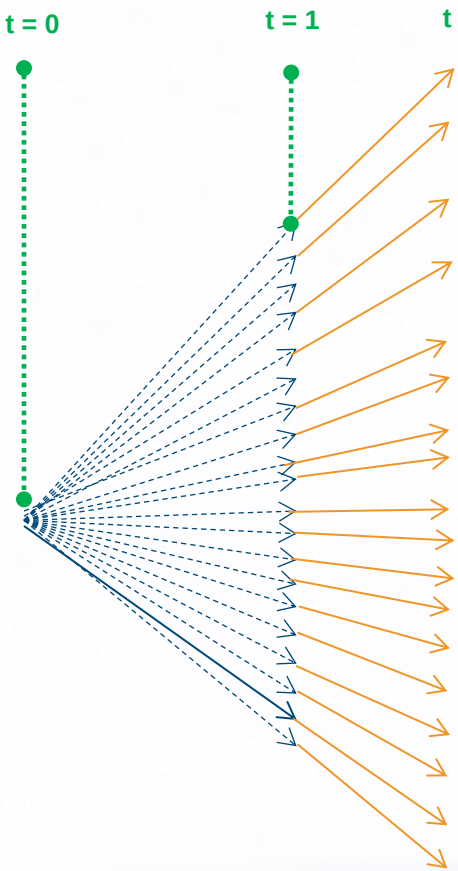
- Scenario set:
  - ***200,000*** risk-neutral interest rate and equity scenarios
  - ***30*** years projection period
  - Calibrated as of ***12/31/2013***.
- The scenarios act as both outer and inner loop. At each point in time along each scenario, the past is outer loop and the future is inner loop. The reserve at a point in time is the PV of future net cash flows.

Case 4: US Variable Annuity – Stochastic on Stochastic

Traditional Nested Stochastic



This Case Study



Least Squares Regression

Milliman

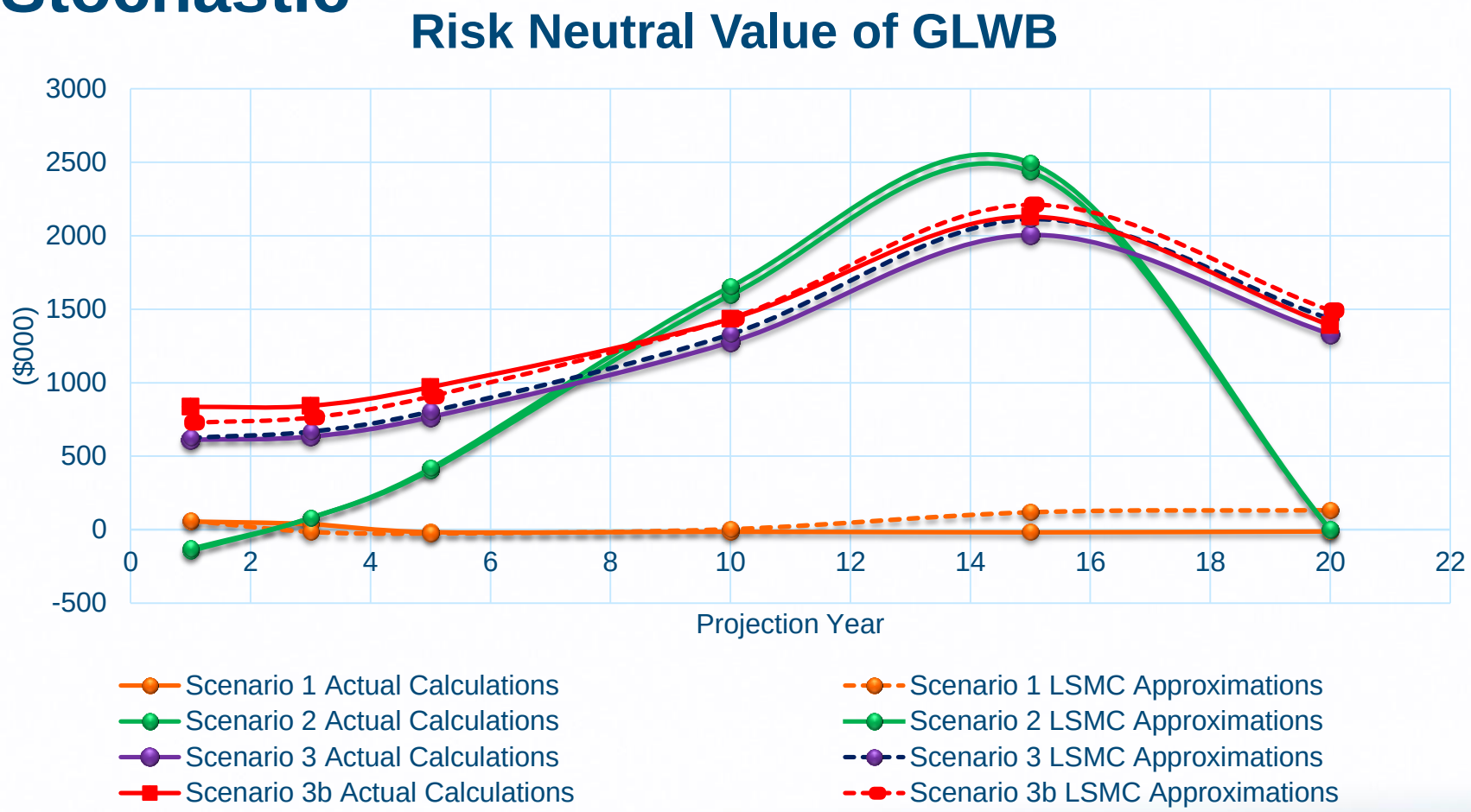
Case 4: US Variable Annuity – Stochastic on Stochastic

Table 1: Difference between Actual Calculation and LSMC Estimate as % of Initial Premium

	Projection Year					
	1	3	5	10	15	20
Scenario 1	-0.1%	0.5%	0.1%	-0.2%	-1.4%	-1.4%
Scenario 2	-0.1%	0.0%	-0.1%	-0.6%	-0.6%	N/a
Scenario 3	-0.1%	-0.3%	-0.4%	-0.5%	-1.1%	-1.0%
Scenario 3b	1.1%	0.8%	0.6%	0.0%	-0.8%	-1.0%
Scenario 4	-0.1%	-0.2%	-0.2%	-0.6%	-0.4%	N/a
Scenario 4b	-1.9%	-1.7%	-1.7%	-1.7%	-0.9%	N/a
Scenario 5	-0.1%	-0.2%	-0.1%	-0.2%	N/a	N/a

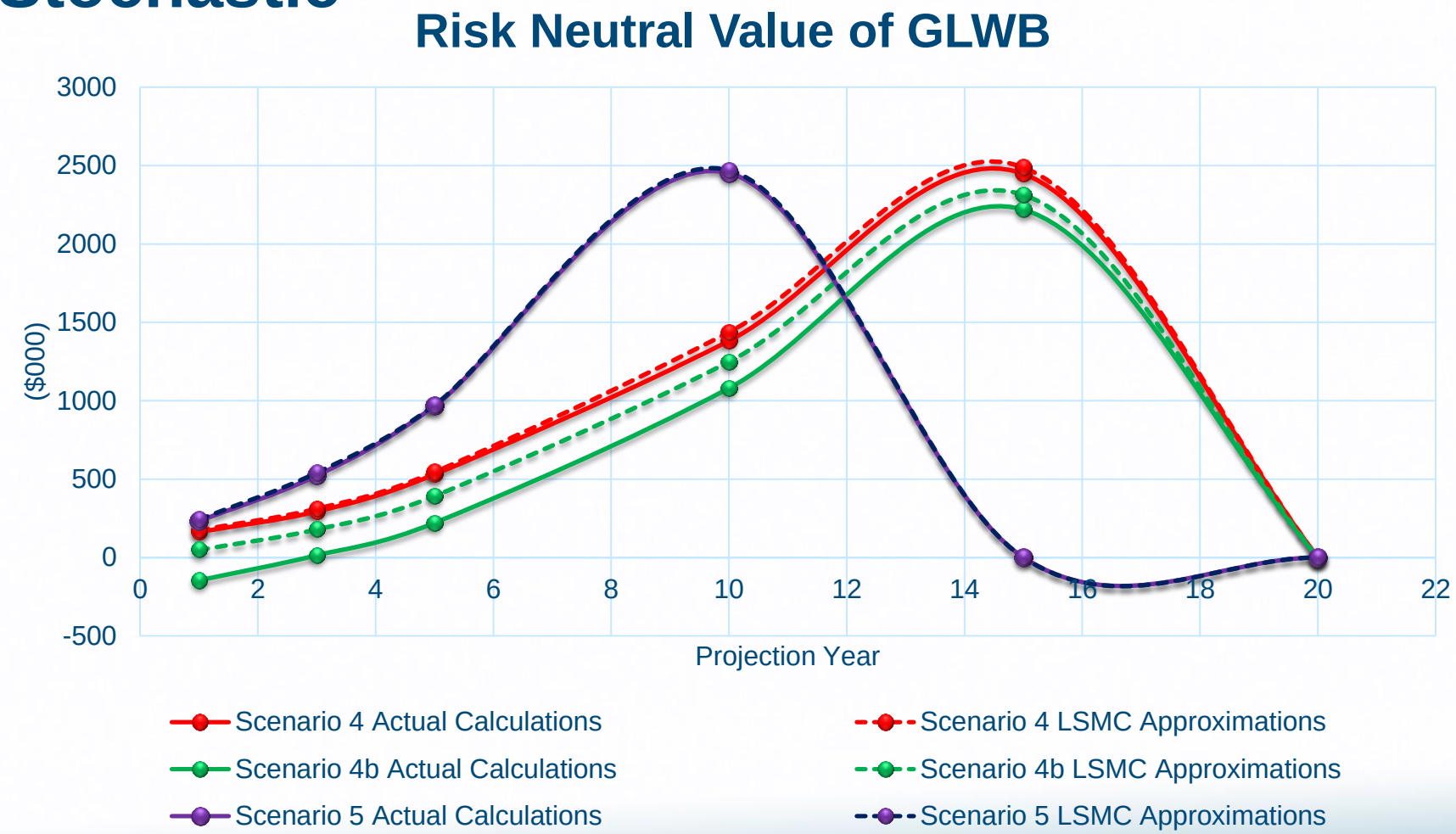
- N/a is shown in periods where no policies remain inforce.
- Scenario 1: Level 8% equity return, 5% bond return
- Scenario 2: 30% equity return in year 1, followed by -5% equity return per year, level 1% bond return
- Scenario 3: -30% equity return in year 1, followed by +5% equity return per year, level 1% bond return
- Scenario 3b: Scenario 3 with 50 bps drop in rates
- Scenario 4: 0% equity and bond return
- Scenario 4b: Scenario 4 with 100 bps increase in rates
- Scenario 5: Level -5% equity return, -1% bond return

Case 4: US Variable Annuity – Stochastic on Stochastic



Milliman

Case 4: US Variable Annuity – Stochastic on Stochastic



Milliman

Final Remarks

- Build good baseline model!
- Make sense of the results, not just statistics!
- Validate
  - Intuition – rule of thumb check
  - Sensitivities at calibration
  - Attribution analysis

Thank you!

David Wang, FIA, FSA, MAAA
Milliman Inc.
1301 Fifth Avenue, Suite 3800
Seattle, WA 98101
david.wang@milliman.com

1-206-504-5524

Milliman