QRM, Extremes, Mathematics and the Financial Crisis

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About the title

I want to discuss with you four concepts:

Concept 1: QRM Concept 2: Extremes (low probability events) Concept 3: Mathematics Concept 4: Financial Crisis

From a (somewhat personal) historical perspective and in the light of the current economic crisis.

(Background: A.J. McNeil, R. Frey and P. Embrechts, Quantitative Risk Management. Concepts, Techniques, Tools. Princeton UP, 2005)

Some relevant examples

- February 1, 1953: Dutch dyke disaster
- January 28, 1985: Challenger explosion
- October 19, 1987: Black Monday
- July 6, 1988: Piper Alpha
- January 17, 1994: Northridge Earthquake
- January 17, 1995: Barings and Kobe
- September 1998: LTCM hedge fund crisis
- 2007-200x (x>9!): Credit crisis
- and (unfortunately) many more ...

Ex.1: 31. Jan. 1953 – 1. Feb. 1953* (February floading)



- 1836 people killed
- 72000 people evacuated
- 49000 houses and farms floaded
- 201000 cattle drowned
- 500 km coastal defenses destroyed; more than 400 breaches of dykes
- 200000 ha land floaded

The Delta – Project

- Coastal fload-protection
- Requested dyke height at I: h_d(I)
- Safety margin at I: MYSS(I) =
 Maximal Yearly Sea Surge at I:



- Probability(MYSS(I) > h_d(I)) should be "small", whereby "small" is defined as: (Risk)
 - 1 / 10000 in the Randstad
 - 1 / 250 in the Deltaregion in the North
 - Similar requirements for rivers, but with 1/10 1/100
- For the Randstad (Amsterdam-Roterdam):
 Dyke height = Normal-level (= NAP) + 5.14 m



NAP

Guus Balkema



Guus Balkema



Laurens de Haan

Springer Series in Operations Research and Financial Engineering

Laurens de Haan Ana Ferreira

Extreme Value Theory An Introduction

Historia Marusata Sin

Guus Balkema

Paul Embrechts

Extremes A geometric approach

High Risk Scenarios and

O Springer

Ex.2: Northridge Earthquake: some loss ratio numbers (%) to think about!



Fin-Ex.1: February 1995







The Great Hanshin (Kobe) earthquake of January 17, 1995



Prime example for Operational Risk, external event (on top of all else)

How Kobe earthquake and a straddle position finally broke the back of Barings bank

Straddle = Short Call and Short Put on Common Strike



Fin-Ex.2: The Black-Scholes Formula(s)

Conditions

$$c = S_0 N(d_1) - K e^{-rT} N(d_2)$$

$$p = K e^{-rT} N(-d_2) - S_0 N(-d_1)$$
where $d_1 = \frac{\ln(S_0 / K) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}}$

$$d_2 = \frac{\ln(S_0 / K) + (r - \sigma^2 / 2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

Financial Derivatives: where (*) it all started (*)

- Black, F., and M. Scholes (1973): "The Pricing of Options and Corporate Liabilities," Journal of Political Economy, 81, 637–654.
- Merton R. C. (1973): "Theory of Rational Option Pricing," Bell Journal of Economics and Management Science, 4, 141–183.
- However (*), L. Bachelier (1900), V. Bronzin (1908), E.O. Thorp (1969)!

The Black-Scholes Model and Model Uncertainty



Just waiting for the storm to hit!

And the Perfect Storm came in September 1998



Nobel







Prize 1997

Some vivid recollections of meetings/discussions with RM and MS:

- 1996: Meetings with RM and MS selling LTCM investment to Swiss Private Banks
- 1996-1998: Several discussions with MS on the use of EVT to calculate (market) regulatory capital: "Who is going to pay for the difference?", i.e. Δ = (VaR-EVT) (VaR-Normal) >> 0
- Cambridge Newton Institute Workshop on Managing Uncertainty, 2001: "Insurance is (just) the other side of the coin (of finance)!" (MS)
- Copula confusion

What about regulation?



- 1988: Basel I
- 1994 2000: Amendment to Basel I (Basel I ½), Value-at-Risk (VaR) is born
- 2000 2009/10(?): Basel II (Credit and Operational Risk)
- Future: Basel III ???
- Later more on this!

What we should have learned from these and similar events:

• One (past) example:

- The Challenger explosion and then more on the current issue:

- The credit crisis ...





ORECAST: MORE FLOODING IN NEW YORK

Lesson 1: A (very) brief discussion on the Challenger explosion



NASA Space Shuttle O-Ring Failures

Richard Feynman

Some key modeling input:

- Logistic regression: theory exists!
- Rare event prediction (31 degr. F)
- Model Uncertainty
- Statistical analysis: data matters!
- Statistical estimation of this uncertainty (95% confidence intervals)

These intervals are typically very wide for

the estimation of rare events

Lesson 2: A (very) brief discussion on the credit crisis

The players (the agents, the components, the jigsaw pieces) (from Crouhy, Jarrow, and Turnbull (2008)):

- Rating Agencies
- Mortgage Brokers and Lenders
- Special Investment Vehicles (SIVs)
- Monolines
- ABS Trust, CDS, CDO and CDO Squared Equity Holders
- Financial Institutions
- The Economy and Central Banks
- Valuation Uncertainty ... once again!!!!
- Transparency, or better... Opaqueness!
- Systemic Risk
- Politicians/the press/lawyers/accountants/the public ...

All of these "components" need a careful and in depth discussion!

As examples of credit derivatives:

CDS = Credit Default Swap
 A relatively simple instrument
CDO = Collateralized Dept Obligation
 A rather complex instrument

A brief technical discussion of the latter and a somewhat more general discussion on the former:

Exhibit 2.9: The conventional wisdom – 2006 (!!!!!)

"There is growing recognition that the dispersion of credit risk by banks to a broader and more diverse group of investors, rather than warehousing such risk on their balance sheets, has helped make the banking and overall financial system more resilient .

The improved resilience may be seen in fewer bank failures and more consistent credit provision. Consequently the commercial banks may be less vulnerable today to credit or economic shocks "

IMF Global Financial Stability Report, April 2006

A stylized Credit Default Swap Set-Up



Securitization

At the heart of the cheap credit binge was a process known as securitization, where Wall Street began to buy up subprime mortgages and package them as mortgage-backed securities to sell to investors. Borrowers

> Mortgage broker

Subprime mortgage lender

More mortgage brokers jumped into the subprime business, many of them earning six-figure incomes on high fees paid for by homeowners'

subprime loans.

Big companies offering subprime mortgages solicited loans nationwide using Wall Street money. After making a home loan. they quickly sold it to packagers, such as investment banks, for more profits.

Structured finance

A financial innovation called structured finance provided Wall Street a way to divide subprime mortgage-backed securities into tranches (French for slices.) The tranches allowed the risk of a loan pool to be parceled out to various investors. Investors who purchased bonds in the securities received a portion of the mortgage payments in the pool.

IOAN

Top-level tranches contain the highest-quality, but lowestpaying, bonds. Even though a

mortgage-backed security may be funded from a pool containing subprime loans, the top tranches can have investmentgrade status of triple-A rated bonds because they are paid first from the pool.

The lowest-level tranches contain the riskiest, highestpaying bonds. They get a low rating and are paid off after the double- and triple-A rated bonds are paid.

Borrowers, many first-time homebuyers or individuals wanting to refinance, turned to subprime loans.

New mortgage loans by year



Wall Street investment banks began pooling risky. subprime loans that did not meet the standards of government-sponsored agencies such as Fannie Mae and sold them as "private label" securities.

Pool of



4

LOAN

CDOs Complexity, Opacity, Distance. Greed, Economic and Political Stupidity, Regulatory Blindness. Academic Naivity, and Arrogance

We are all to blame!

The Denver Post

Mortgage-

Investors worldwide

securities.

aobbled up the

backed

6

security

A sure road for disaster

- In the previous picture, problems occur if several corporations default at the same time, in that case the insurance companies have to pay, may loose their high rating causing the pension funds (investors) more problems, etc, etc ... someone at some time will blame the rating agencies
- But what about the hedge funds ... ?
- In the end all depends on default correlation ... enters the Gauss-copula.

Impact of dependence on loss distribution



Distribution of number of defaults for homogeneous portfolio of 1000 BB loans with default probability $\approx 1\%$; Bernoulli mixture model with default correlation $\approx 0.22\%$ is compared with independent default model.

CDOs - Basic Structure

There are a variety of CDO contracts, but all have the same basic structure. Each CDO has a asset side, and a liability side, linked by a special purpose vehicle (SPV).

- The assets consist of credit risky securities related to a pool of reference entities; typically bonds, loans or - in synthetic CDOs - a protection-seller positions in single name CDS.
- These assets are acquired by the SPV. To finance the asset purchase the SPV issues notes. This amounts to a repackaging of the assets.
- The notes form the liability side of the structure. They belong to tranches of different seniority, called senior, mezzanine and equity piece. Due to repackaging most losses of the assets are borne by the equity piece, and the credit rating of mezzanine and senior tranches is higher than average rating of asset pool.

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(Synthetic) CDOs - Basic Structure



Synthetic CDOs: Payment Description

Notation. Consider portfolio of m loans with nominal e_i , relative LGD δ_i and default-indicator process (\mathbf{Y}_t) . Cumulative loss of the portfolio in t given by $L_t = \sum_{i=1}^m \delta_i e_i Y_{t,i}$.

The CDO. Maturity T. We have k tranches, characterized by attachment points $0 = K_0 < K_1 < \cdots < K_k \leq \sum_{i=1}^m e_i$. The notional of tranche κ at time t is given by

$$N_{\kappa}(t) = f_{\kappa}(L_t) \text{ with } f_{\kappa}(l) = \begin{cases} K_{\kappa} - K_{\kappa-1} & \text{ for } l < K_{\kappa-1} \\ K_{\kappa} - l & \text{ for } l \in [K_{\kappa-1}, K_{\kappa}]. \\ 0 & \text{ for } l > K_{\kappa} \end{cases}$$

Note that $f_{\kappa}(l) = (K_{\kappa} - l)^{+} - (K_{\kappa-1} - l)^{+}$ (put spread with strike prices K_{κ} and $K_{\kappa-1}$).

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Payments of a Synthetic CDO

Consider CDO with attachment points $K_0 < \cdots < K_k$ and notional of tranche κ given by $N_{\kappa}(t) = (K_{\kappa} - L_t)^+ - (K_{\kappa-1} - L_t)^+$; define cumulative loss of tranche κ as $L_{\kappa}(t) := N_{\kappa}(0) - N_{\kappa}(t)$.

Default payments of CDO. Default payment of tranche κ at nth default time $T_n < T$ given by $\Delta L_{\kappa}(T_n) = (L_{\kappa}(T_n) - L_{\kappa}(T_{n-1}))$ (the part of cumulative loss at T_n falling in the layer $[K_{\kappa-1}, K_{\kappa}]$).

Protection fee or premium payments. Holder of tranche κ receives periodic premium payments at $0 < t_1 < \cdots < t_N = T$ of size $x_{\kappa}^{\text{CDO}}(t_n - t_{n-1})N_{\kappa}(t_n)$. No initial payments. x_{κ}^{CDO} is called the (fair) CDO spread.

A stylized Example

Stylized CDO. We assume that payoff of tranche κ is simply given by $N_{\kappa}(T)$, the value of the notional at maturity. Real CDOs are more complicated, as there is intermediate income, but stylized example retains essential features.

Impact of default dependence. More dependence, same marginal default probabilities \Rightarrow Equity tranche increases in value, senior tranches decrease in value. Impact on metazanine tranches unclear.

Qualitative properties carry over to more complex structures actually traded.

Default Correlation and CDO Tranches

The waterfall principle



Payoff of a stylized CDO with attachment points at 20, 40 and 60 with two different loss distributions overlayed.

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An import critical voice:

Warren Buffet on Derivatives (Berkshire Hathaway annual report for 2002):

The derivatives genie is now well out of the bottle, and these instruments will almost certainly multiply

in variety and number until some event makes their toxicity clear. Central banks and governments have so far found no effective way to control, or even monitor, the risks posed by these contracts. In my view, derivatives are financial weapons of mass destruction, carrying dangers that, while now latent, are potentially lethal.



Some dimensions before we continue:

Thousand \$ = 1 000 \$ Million \$ = 1 000 000 \$ Billion (US) \$ = 1 000 000 000 \$ ____ = 1 Milliard (UK) \$ Trillion (US) \$ = 1 000 000 000 000 \$ ----= 1 Billion (UK)= 1 Billion \$ (Germany) Trillion (UK) \$ = 1 000 000 000 000 000 000 \$

Everything clear ... I hope.

50 000 000 000 000 \$ *

- CDS is almost a brand new investment vehicle, but the market is already 20 times its size in 2000. The principal amount of CDS outstanding equals \$50 trillion, or more than three times the U.S. Gross Domestic Product and bigger than all the U.S. credit markets put together. And the CDS has been a huge source of "financial engineering" profits, both for Wall Street and the hedge fund community over the last few years.
- World GDP is about \$66 trillion.
- First CDS about 1995.
- Total nominal volume of OTC derivatives 550 Tri. \$

* 3.7 Tri. \$ after netting

The basic copula construction (d = 2)



Distribution Functions A-B





Distribution Functions C-D



Uniform density



David X. Li (2000) On Default Correlation: A Copula

Function Approach, Journal of Fixed Income 9:43-54

- This paper studies the problem of default correlation. We first introduce a random variable called "time-until- default" to denote the survival time of each defaultable entity or financial instrument, and define the default correlation between two credit risks as the correlation coefficient between their survival times. Then we argue why a copula function approach should be used to specify the joint distribution of survival times after marginal distributions of survival times are derived from market information, such as risky bond prices or asset swap spreads. The definition and some basic properties of copula functions are given. We show that the current CreditMetrics approach to default correlation through asset correlation is equivalent to using a normal copula function. Finally, we give some numerical examples to illustrate the use of copula functions in the valuation of some credit derivatives, such as credit default swaps and first-to-default contracts.
- netion approach to credit, we start

• April 1, 2000 (sic)

David Li 8 years later

(The Gauss-copula)

The Gauss-Copula (d=2)

In the two-dimensional case with correlation parameter p

$$C^{\Phi}(a,b) = \Phi_2(\Phi^{-1}(a), \Phi^{-1}(b); \varrho) = \int_{-\infty}^{\Phi^{-1}(a)} \int_{-\infty}^{\Phi^{-1}(b)} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(\frac{2\rho uv - u^2 - v^2}{2(1-\rho^2)}\right) du \, dv$$

And with density function

$$c^{\Phi}(a,b) = \frac{\exp\left(\frac{(\Phi^{-1}(a))^2 + \Phi^{-1}(b))^2}{2} + \frac{2\rho\Phi^{-1}(a)\Phi^{-1}(b) - (\Phi^{-1}(a))^2 - (\Phi^{-1}(b))^2}{2(1-\rho^2)}\right)}{\sqrt{1-\rho^2}}$$

Let us call the Gauss-Copula the normal-copula!

Two results from the 1998 RiskLab report

CORRELATION AND DEPENDENCE IN RISK MANAGEMENT: PROPERTIES AND PITFALLS

PAUL EMBRECHTS, ALEXANDER MCNEIL, AND DANIEL STRAUMANN

Remark 1: See Figure 1 next page

A very early warning!

1959

Remark 2: In the above paper it is shown that

Thus the Gaussian copula gives asymptotic independence, provided that $\rho < 1$. Regardless of how high a correlation we choose, if we go far enough into the tail, extreme events appear to occur independently in each margin. See Sibuya (1961) or Resnick (1987), Chapter 5, for alternative demonstrations of this fact.



FIGURE 1. 1000 random variates from two distributions with identical Gamma(3,1) marginal distributions and identical correlation $\rho = 0.7$, but *different* dependence structures.

The Gauss-copula model caused a first strong breeze:



September 12, 2005

How a Formula Ignited Market That Burned Some Big Investors

The model Mr. Li devised helped estimate what return investors in certain credit derivatives should demand, how much they have at risk and what strategies they should employ to minimize that risk. Big investors started using the model to make trades that entailed giant bets with little or none of their money tied up. Now, hundreds of billions of dollars ride on variations of the model every day.

"David Li deserves recognition," says Darrell Duffie, a Stanford University professor who consults for banks. He "brought that innovation into the markets [and] it has facilitated dramatic growth of the credit-derivatives markets."

David Li warned himself early on:



The problem: The scale's calibration isn't foolproof. "The most dangerous part," Mr. Li himself says of the model, "is when people believe everything coming out of it." Investors who put too much trust in it or don't understand all its subtleties may

think they've eliminated their risks when they haven't.

David X. Li in Wall Street Journal Article, 2005.

But then the Perfect Storm struck (again)!













And once more, the popular press blamed the mathematicians, the quants, through the Gaussian-(normal-)copula, for having blown up the economy!

Numerous newspaper articles:

- Felix Salmon, 23 February, 2009, Wired Magazine (a web-blog): Recipe for Disaster: The Formula That Killed Wall Street
- The Financial Times, Sam Jones (April 24, 2009), On Couples and Copulas
- Steve Lohr, September 12, 2009, NY Times, Wall Street's Math Wizards Forgot a Few Variables

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Recipe for Disaster: The Formula That Killed Wall Street By Felix Salmon 23 February, 2009 Wired Magazine



The popular press is full of statements like:

- From risk-free return to return-free risk
- Mark-to-market, mark-to-model, mark-to-myth
- Here's what killed your 401(k)
- Mea Copula
- Anything that relies on correlation is charlatanism (N.N.Taleb)
- Double defeat for Wall Street and Mathematics
- Rather than common sense, financial mathematics was ruling
- Etc ...

Even the Financial Times joins in:

Of couples and copulas by Sam Jones (April 24, 2009)

In the autumn of 1987, the man who would become the world's most influential actuary landed in Canada on a flight from China. He could apply the broken hearts maths to broken companies.

Li, it seemed, had found the final piece of a riskmanagement jigsaw that banks had been slowly piecing together since quants arrived on Wall Street.

Why did no one notice the formula's Achilles heel?



Johnny Cash and June Carter

Dear Sir

The article "Of couples and copulas", published on 24 April 2009, suggests that David Li's formula is to blame for the current financial crisis. For me, this is akin to blaming Einstein's E=mc² formula for the destruction wreaked by the atomic bomb.

Feeling like a risk manager whose protestations of imminent danger were ignored, I wish to make clear that many well-respected academics have pointed out the limitations of the mathematical tools used in the finance industry, including Li's formula. However, these warnings were either ignored or dismissed with a desultory response: "It's academic".

We hope that we are listened to in the future, rather than being made a convenient scapegoat.

Yours Faithfully, Professor Paul Embrechts Director of RiskLab ETH Zurich

Some personal recollections on the issue:

28 March 1999

Columbia-JAFEE Conference on the Mathematics of Finance,

Columbia University, New York.

10:00-10:45 P. EMBRECHTS (ETH, Zurich):

"Insurance Analytics:

Actuarial Tools in Financial Risk-Management"

Why relevant?

1. Paper: P. Embrechts, A. McNeil, D. Straumann (1999) Correlation and Dependence in Risk Management: Properties and Pitfalls. Preprint RiskLab/ETH Zürich.

2. Coffee break: discussion with David Li.

If you are interested in my views on Copulas and QRM:

- Read my paper:
 - "Copulas: A personal view"
 - Journal of Risk and Insurance, 2009.
 - See also my website:
 - www.math.ethz.ch/~embrechts

There were however several early warnings (1) It Doesn't Take Nostradamus

JOSEPH E. STIGLITZ

Economists' Voice: <u>www.bwpress.com/ev</u> November, 2008

"I went on to explain how securitization can give rise to perverse incentives ... Has the growth in securitization been result of more efficient transactions technologies, or an unfounded reduction in concern about the importance of screening loan applications? ... we should at least entertain the possibility that it is the latter rather than the former."

At the very least, the banks have demonstrated an ignorance of two very basic aspects of risk: (a) the importance of correlation, and (b) the possibility of price decline.

REFERENCES AND FURTHER READING

Stiglitz, Joseph (1992) "Banks versus Markets as Mechanisms for Allocating and Coordinating Investment," in J. Roumasset and S. Barr (ed.) The Economics of Cooperation. Boulder: Westview Press, Inc. (Paper originally presented at a conference at the University of Hawaii, January, 1990.) There were however several early warnings (2)

Embrechts, P. et al. (2001): An academic response to Basel II. Financial Markets Group, London School of Economics. (Mailed to the Basel Committee)

(Critical on VaR, procyclicality, systemic risk)









There were however several early warnings (3)

Markopolos, H. (2005): The world's largest hedge fund is a fraud. (Mailed to the SEC)

(Madoff runs a Ponzi scheme)



Harry Markopolos





Bernard Madoff

Charles Ponzi 1910 The Turner Review A regulatory response to the global banking crisis March 2009, FSA, London (126 pages)

1.1 (iv) Misplaced reliance on sophisticated maths

There are, however, fundamental questions about The validity of VAR as a measure of risk (see Section 1.4 (ii) below). And the use of VAR measures based on relatively short periods of historical observation



(e.g. 12 months) introduced dangerous procyclicality into the assessment of tradingbook risk for the reasons set out in Box 1A (deficiencies of VAR).

The very complexity of the mathematics used to measure and manage risk, moreover, made it increasingly difficult for top management and boards to assess and exercise judgement over the risks being taken. Mathematical sophistication ended up not containing risk, but providing false assurance that other prima facie indicators of increasing risk (e.g. rapid credit extension and balance sheet growth) could be safely ignored.

1.1 (v) Hard-wired procyclicality: ...

1.4 (iii) Misplaced reliance on sophisticated maths: fixable deficiencies or inherent limitations?

Four categories of problem can be distinguished:

- Short observation periods
- Non-normal distributions
- Systemic versus idiosyncratic risk
- Non-independence of future events; distinguishing risk
 and uncertainty

Frank H. Knight, 1921

This is the main reason why we make a difference between Model Risk and Model Uncertainty. We very much stress the latter!

Supervisory guidance for assessing banks' financial instrument fair value practices

April 2009, Basel Committee on Banking Supervision

• Principle 8: Supervisors expect bank valuation and risk measurement systems to systematically recognise and account for valuation uncertainty. In particular, valuation processes and methodologies should produce an explicit assessment of uncertainty related to the assignment of value for all instruments or portfolios. When appropriate this may simply be a statement that uncertainty for a particular set of exposures is very small. While qualitative assessments are a useful starting point, it is desirable that banks develop methodologies that provide, to the extent possible, quantitative assessments. These methodologies may gauge the sensitivity of value to the use of alternative models and modelling assumptions (when applicable), to the use of alternative values for key input parameters to the pricing process, and to alternative scenarios to the presumed availability of counterparties. The extent of this analysis should be commensurate to the importance of the specific exposure for the overall solvency of the institution.

Financial Mathematics and the Credit Crisis

"If Financial Mathematicians have an understanding of the derivative products at the root of the credit crisis, can they offer any insights on the current economic situation. Specifically, there is a sense of gloom that "The City is over" and is there a more positive view."

(Question posed to researchers by Lord Drayson, the UK Science and Innovation Minister)

Some replies by researchers:

 (L.C.G. Rogers) The problem is not that mathematics was used by the banking industry, the problem was that it was abused by the banking industry. Quants were instructed to build models which fitted the market prices. Now if the market prices were way out of line, the calibrated models would just faithfully reproduce those wacky values, and the bad prices get reinforced by an overlay of scientific respectability!

And Rogers continues:

- The standard models which were used for a long time before being rightfully discredited by (some) academics and the more thoughtful practitioners were from the start a complete fudge; so you had garbage prices being underpinned by garbage modelling.
- (M.H.A. Davis) The whole industry was stuck in a classic positive feedback loop which no party could (P.E. "wanted to") walk away from.

Indeed only some!

EVT = Extreme Value Theory

EVT (first established around the 1920's) offers a sound set of techniques for the understanding and statistical estimation of rare events, beyond the bell-curve world: it describes the statistical behavior of the largest observation, the biggest loss, the worst case, rather than the average observation, the average loss, the average case.

For details: see the following textbooks!

EVT has become a standard

Some examples (but there are many more)



A message for my students

New generations of students will have to use the tools and techniques of QRM wisely in a world where the rules of the game will have been changed.

Always be scientifically critical, as well as socially honest, adhere to the highest ethical principles, especially in the face of temptation ... which will come! And on the boundedness of our knowledge:

There are more things in heaven and earth, Horatio, than are dreamt of in your philosophy!

William Shakespeare (Hamlet I.v. 166)