Dilution effects of executive stock option awards

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ABSTRACT

A closed-form approximation is developed for stock dilution resulting from the implementation of multiple-tranche executive stock option programs. Measurements of the sensitivity of dilution to underlying parameters and its impact on estimated earnings per share and increased numbers of shares are also quantified.

1. Introduction

As a part of their compensation to executives, firms use employee stock options more frequently and in larger amounts than in past decades. Many research studies have been carried out to establish a methodology for valuing these executive stock options (e.g., Hemmer *et al* 1994, Huddart and Lang 1996, Hall 1999, Johnson and Tian 2000). Also, the Financial Accounting Standards Board has established guidelines (e.g., FASB-123) for estimating the fair value of such options and requires corporations to report them in their financial statements. The growing use across all firms of executive stock options has drawn intense scrutiny from the investment community, including analysts, regulators and individual and institutional investors, with particular concern for the dilution of earnings per share (EPS) that results from the issuance of additional shares by corporations when executive stock options are exercised.

For a summary assessment of firms' anticipated performance, expected EPS has been singled out as a key measure (Frankel and Lee 1998: Dechow *et al* 1999). The evaluation of EPS requires two components, company earnings and the number of shares issued. As earnings are likely to be predicted based on many factors independent of shares outstanding, the focus of this study is the denominator of the EPS term. Dilution, a commonly understood concept but without a specific universally accepted measure, is defined here as the fraction of ownership represented by one share of outstanding stock (equivalently, the reciprocal of the number of shares issued and outstanding). Dilution is affected by the issuance of additional company shares that results from the exercise of executive stock options. There have been some empirical studies to examine the dilution factor in companies but there is no available methodology for dilution estimation that incorporates the effects of various types of executive stock option plans. This paper develops a workable formula to estimate the expected dilution due to executive stock option plans which consists of multiple-issue tranches. We derive formulas for the dilution and its impact on EPS and number of shares outstanding. The dilution formula derived is analogous to the well-known Black -

Scholes formula and can be easily employed in practical situations. We also examine the influence on dilution of other factors in the firm, including its sensitivity to changes in stock price. Finally, we provide some numerical results on the behavior of dilution as some of the independent parameters vary.

2. Expected dilution and its sensitivity

Consider a series of executive stock option awards which are exercisable at times $\tau_1, \tau_2, ..., \tau_m$ with respective exercise prices $K_1, K_2, ..., K_m$ .The respective numbers of shares covered by the executive stock options are taken to be $n_1, n_2, ..., n_m$, with $n_i << N_0$, the number of shares of the firm at time t=0. For convenience, we take $n_0 = 0$. We assume that the stock price process has a Geometric Brownian motion with the stock price at time t given by $S(t) = S(0)\exp\{X(t)\}$, where X (t) is normal with mean and variance per unit time μ and σ^2 respectively. Consider the dilution that would occur at time τ_1, the first time when executive stock options may be exercised. Let $k_i = \ln(K_i / S(0))$ and $X_i = X(\tau_i)$, i=1,2,…,m. Then, when an ESO is exercised for one share at time τ_1, the net additional fraction of a share to be issued is $1 - \frac{K_1}{S(\tau_1)} = 1 - \frac{K_1}{S(0)} e^{-X_1}$, assuming that the exercise price is used to fund share repurchases. Let N_i denote the (random) number of outstanding shares in the company just after the exercise time t=τ_i. Then N_1 is given by

$$N_1 = N_0 \text{ if } X_1 \le k_1 \text{, or}$$

$$N_1 = N_0 + n_1(1 - \frac{K_1}{S(0)} e^{-X_1}) \text{ if } X_1 > k_1. \tag{2.1}$$

Let $D_i = 1/N_i$ denote the dilution at time τ_i and define the event $A_i = \{X_i \le k_i\}$ for i=1,2,…,m. Then

$$D_1 = \frac{I(A_1)}{N_0} + \frac{I(\overline{A}_1)}{[N_0 + n_1(1 - e^{-X_1 + k_1})]}, \tag{2.2}$$

where $I(A)$ is the indicator function for the event A and $\overline{A}$ is the complementary event. By expanding the second term in (2.2) as a power series and neglecting terms of order $(\frac{n_1}{N_0 + n_1})^2$ and higher, consistent with the assumption that $n_i << N_0$, we have, as an approximation,

$$D_1 = \frac{1}{N_1} = \frac{1}{N_0} - \frac{n_1 I(\overline{A}_1)}{N_0(N_0+n_1)} + \frac{n_1 e^{-X_1+k_1} I(\overline{A}_1)}{(N_0+n_1)^2}. \tag{2.3}$$

In the following, all equations for dilution and expected dilution express high-order approximations similar to those used for D_1 above. Taking expectation of (2.3) with respect to the risk-neutral measure, it follows that

$$E_Q(D_1) = E(D_0) - \frac{n_1}{N_0(N_0+n_1)} E_Q[I(X_1 > k_1)] + \frac{n_1 e^{k_1}}{(N_0+n_1)^2} E_Q[e^{-X_1} I(X_1 > k_1)] \tag{2.4}$$

with $E(D_0) = \dfrac{1}{N_0}$.

Now, using the Esscher transformation (Gerber and Shiu 1994), $E_Q[I(X_1 > k_1)] = E[I(X_1 > k_1); h]$ where h is the Esscher transform parameter. The random variable X_1 under Esscher transform has mean per unit time given by $\mu + h\sigma^2 = r - \sigma^2/2$ and variance σ^2 per unit time. It follows that

$$E_Q[I(X_1 > k_1)] = \Phi(-\frac{k_1 - (r-\sigma^2/2)\tau_1}{\sigma\sqrt{\tau_1}}) \tag{2.5}$$

where $\Phi(.)$ is the distribution function of the standard normal random variable.

Using a similar computation for the other expectation in (2.4), we find that

$$\begin{aligned} E_Q(D_1) &= E(D_0) - \frac{n_1}{N_0(N_0+n_1)} \Phi(-\frac{k_1 - (r-\sigma^2/2)\tau_1}{\sigma\sqrt{\tau_1}}) \\ &+ \frac{n_1 K_1 e^{(\sigma^2 - r)\tau_1}}{S(0)(N_0+n_1)^2} \Phi(-\frac{k_1 - (r-3\sigma^2/2)\tau_1}{\sigma\sqrt{\tau_1}}). \end{aligned} \tag{2.6}$$

We next consider the situation at time t=τ_2. Given N_1, we have

$$N_2 = N_1 + n_2(1 - e^{-X_2+k_2}) I(\overline{A}_2). \tag{2.7}$$

Using the same procedure as in the derivation of (2.3), with terms of order $(\dfrac{n_2}{N_1+n_2})^2$ and higher neglected, we have

$$D_2 = \frac{1}{N_1} - \frac{n_2 I(\overline{A}_2)}{N_1(N_1+n_2)} + \frac{n_2 e^{-X_2+k_2} I(\overline{A}_2)}{(N_1+n_2)^2}. \tag{2.8}$$

Combining (2.3) and (2.8), it follows that

$$D_2 = \frac{1}{N_0} - \frac{n_1 I(\overline{A}_1)}{N_0(N_0+n_1)} + \frac{n_1 e^{-X_1+k_1} I(\overline{A}_1)}{(N_0+n_1)^2} - \frac{n_2 I(\overline{A}_2)}{N_1(N_1+n_2)} + \frac{n_2 e^{-X_2+k_2} I(\overline{A}_2)}{(N_1+n_2)^2} \quad (2.9)$$

Making note of the fact that

$$\frac{1}{(N_1+n_2)} = \frac{1}{(N_0+n_1+n_2)}[1+\frac{n_1(1-e^{-X_1+k_1} I(\overline{A}_1))}{(N_1+n_2)}] \quad (2.10)$$

and neglecting terms of higher order as before, (2.9) is then reduced to

$$D_2 = \frac{1}{N_0} - \frac{n_1 I(\overline{A}_1)}{N_0(N_0+n_1)} + \frac{n_1 e^{-X_1+k_1} I(\overline{A}_1)}{(N_0+n_1)^2} - \frac{n_2 I(\overline{A}_2)}{(N_0+n_1)(N_0+n_1+n_2)} + \frac{n_2 e^{-X_2+k_2} I(\overline{A}_2)}{(N_0+n_1+n_2)^2}. \quad (2.11)$$

Taking expectation of (2.11) with respect to the risk-neutral measure, we have

$$E_Q(D_2) = \frac{1}{N_0} - \frac{n_1}{N_0(N_0+n_1)}\Phi(-\frac{k_1-(r-\sigma^2/2)\tau_1}{\sigma\sqrt{\tau_1}}) + \frac{n_1 K_1 e^{(\sigma^2-r)\tau_1}}{S(0)(N_0+n_1)^2}\Phi(-\frac{k_1-(r-3\sigma^2/2)\tau_1}{\sigma\sqrt{\tau_1}})$$
$$-\frac{n_2}{(N_0+n_1)(N_0+n_1+n_2)}\Phi(-\frac{k_2-(r-\sigma^2/2)\tau_2}{\sigma\sqrt{\tau_2}}) + \frac{n_2 K_2 e^{(\sigma^2-r)\tau_2}}{S(0)(N_0+n_1+n_2)^2}\Phi(-\frac{k_2-(r-3\sigma^2/2)\tau_2}{\sigma\sqrt{\tau_2}}). \quad (2.12)$$

Using the same arguments as for (2.6) and (2.12) and recalling that $n_0 = 0$, one can express the expected value of the dilution at the final exercise time t= τ_m of all *m* tranches of ESOs as

$$E_Q(D_m) = \frac{1}{N_0} - \sum_{i=1}^{m} \frac{n_i}{(N_0 + \sum_{j=0}^{i} n_j)} \{ \frac{\Phi(d_1(i))}{(N_0 + \sum_{j=0}^{i-1} n_j)} - \frac{K_i e^{(\sigma^2-r)\tau_i}\Phi(d_2(i))}{S(0)(N_0 + \sum_{j=0}^{i} n_j)} \}, \quad (2.13)$$

where $d_1(i) = \frac{(r-\sigma^2/2)\tau_i - k_i}{\sigma\sqrt{\tau_i}}$ and $d_2(i) = d_1(i) - \sigma\sqrt{\tau_i}$.

To establish an estimate for EPS at the final exercise time τ_m, for example, one can multiply the estimated total earnings by the term $E_Q(D_m)$ given in (2.13), assuming independence of earnings and dilution.

Sensitivity of dilution

The impact on the dilution due to a change in one of the parameters is also useful in measuring its sensitivity (the “Greeks”).

We first derive the sensitivity of dilution to changes in the initial stock price $S(0)$ in the case of a single exercise time (m=1). This is given by $\frac{\partial}{\partial S(0)} E_Q(D_1)$ which, using (2.6), is

$$\frac{\partial}{\partial S(0)} E_Q(D_1) = -\frac{n_1 K_1 e^{(\sigma^2 - r)\tau_1}}{S(0)^2 (N_0 + n_1)^2} \Phi(d_2(1)) + \frac{n_1}{(N_0 + n_1) S(0) \sigma \sqrt{\tau_1}} [\frac{K_1 e^{(\sigma^2 - r)\tau_1}}{S(0)(N_0 + n_1)} \phi(d_2(1)) - \frac{1}{N_0} \phi(d_1(1))] \quad (2.14)$$

where $\phi(.)$ is the density of the standard normal random variable.

We note that

$$\phi(d_2(1)) = \phi(d_1(1)) \exp\{-k_1 + (r - \sigma^2)\tau_1\}. \quad (2.15)$$

It follows, using (2.15) in (2.14), that

$$\frac{\partial}{\partial S(0)} E_Q(D_1) = -\frac{n_1 K_1 e^{(\sigma^2 - r)\tau_1}}{S(0)^2 (N_0 + n_1)^2} \Phi(d_2(1)) - \frac{{n_1}^2 \phi(d_1(1))}{(N_0 + n_1)^2 N_0 S(0) \sigma \sqrt{\tau_1}}. \quad (2.16)$$

Neglecting terms of higher order in n_1 / N_0, the sensitivity (Δ) of dilution to stock price in the case m=1 is

$$\Delta = \frac{\partial}{\partial S(0)} E_Q(D_1) = -\frac{n_1 K_1 e^{(\sigma^2 - r)\tau_1}}{S(0)^2 (N_0 + n_1)^2} \Phi(d_2(1)), \quad (2.17)$$

indicating that for each small increase in stock price at the time of award of ESOs, the expected dilution decreases by a factor given by (2.17). That is, a rising stock price reduces the fractional ownership of each company share.

We can show in the same manner that, when m>1, we have

$$\Delta = \frac{\partial}{\partial S(0)} E_Q(D_m) = -\sum_{i=1}^{m} \frac{K_i n_i e^{(\sigma^2 - r)\tau_i}}{S(0)^2 (N_0 + \sum_{j=1}^{i} n_j)^2} \Phi(d_2(i)). \quad (2.18)$$

Similarly the sensitivity of dilution to the exercise time may be evaluated. With m=1, the appropriate derivative is

$$\frac{\partial}{\partial \tau_1} E_Q(D_1) = \frac{n_1 K_1 (\sigma^2 - r) e^{(\sigma^2 - r)\tau_1}}{S(0)(N_0 + n_1)^2} \Phi(d_2(1))$$
$$- \frac{n_1 \sigma \phi(d_1(1))}{2(N_0 + n_1)^2 \sqrt{\tau_1}} - \frac{n_1^2 \phi(d_1(1))}{2N_0(N_0 + n_1)^2} [(\frac{r}{\sigma} - \frac{\sigma}{2})\tau_1^{-.5} + \frac{k_1}{\sigma}\tau_1^{-1.5}]. \quad (2.19)$$

However, the last term in (2.19) is negligible in light of the assumption that $n_1 << N_0$ and hence the sensitivity of the dilution to exercise time is given as another one of the Greeks by

$$\Theta = -\frac{\partial}{\partial \tau_1} E_Q(D_1) = -\frac{n_1}{(N_0 + n_1)^2} (\frac{K_1(\sigma^2 - r)e^{(\sigma^2 - r)\tau_1}\Phi(d_2(1)}{S(0)} - \frac{\sigma\phi(d_1(1))}{2\sqrt{\tau_1}}). \quad (2.20)$$

3. Expected number of shares and fractional ownership

Suppose we are interested in evaluating the expected fractional change in percentage ownership due to an ESO award when the number of shares outstanding increases from N_0 to $N_0 + \Delta N$ at a time t.

Assuming that the company's tangible value is the same, the per-share value after the increase is $\frac{1}{(1+\frac{\Delta N}{N_0})}$ times the per-share value before the increase.

In this case the fractional increase in shares is $\frac{1}{N_0}$ times the net increase in shares resulting from exercise.

Using the work in Section 2, the expected fractional increase in shares due to a series of ESO awards is given by

$$E_Q(\sum_{i=1}^{m} \Delta N_i / N_0) = \frac{1}{N_0 S(0)} \sum_{i=1}^{m} n_i K_i e^{(\sigma^2 - r)\tau_i} \Phi(d_2(i)) \ . \quad (3.1)$$

4. Numerical Illustrations and Discussion

An illustration of the sensitivity of dilution to changes in stock price is given in Figure 1. For this example, the parameter values are chosen to be $N_0 = 10{,}000{,}000$, $n_i = .01N_0$, r=.04, m=1, $\tau_1 = 1$, $\sigma = .4$ and $K_i = 4$, with values of

S_0 from 1 to 10. As indicated by (2.18), the dilution multiplier (the multiple of the original share number that gives the number of shares after exercise) increases, so that fractional share ownership decreases with an increase in stock price. Thus, as ESOs are priced more "out of the money," there are relatively fewer shares that can be repurchased with the proceeds, contributing to an increased total number of shares.

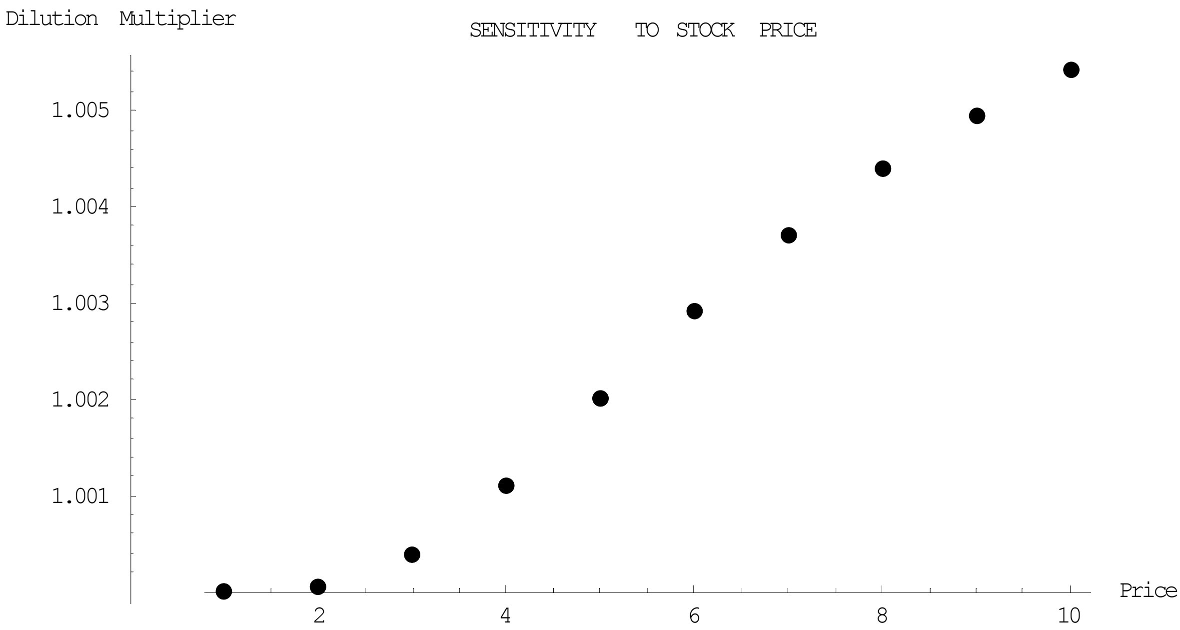

Figure 1

In Figure 2, the sensitivity of dilution to changes in option exercise time is illustrated, with parameters $N_0 = 10{,}000{,}000$, $n_i = .01N_0$, r=.04, m=1, $\sigma = .4$, $K_i = 1.1S_0$ and S_0=40, with exercise time τ_1 ranging from 1 to 10 years. In this case, the dilution multiplier again increases (and fractional share ownership decreases) as the exercise time is advanced. For other values of r and σ, the dilution multiplier may decrease. It should come as no surprise that the impact of the ESO's levels off as the exercise time becomes infinite when other parameters are unchanged.

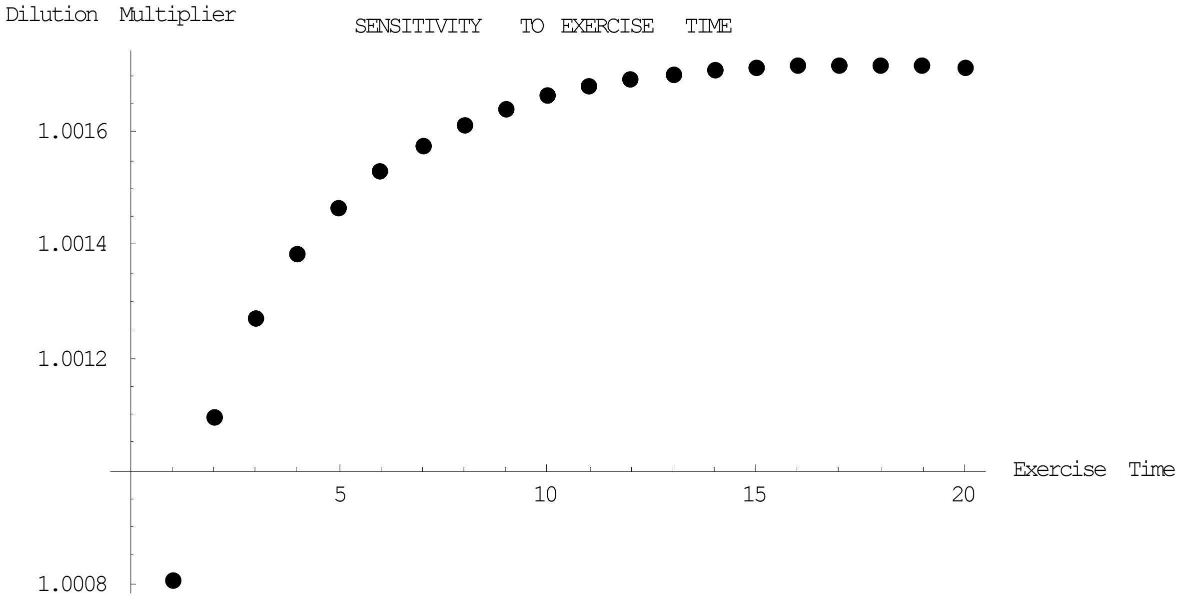

Figure 2

As hinted at above, some surprises can occur when the sensitivity of dilution to stock volatility is considered. Figure 3 depicts the dilution multiplier as a function of the volatility σ with values of σ between 0.1 and 4.0 when the other parameters are $N_0 = 10{,}000{,}000$, $n_i = .01N_0$, r=.04, m=1, S_0=40, $K_i = 1.1S_0$ and $\tau_1 = 1$. In this case the dilution multiplier peaks for a value of σ near .5 and decreases toward 1 as the shares become increasingly volatile. One finding of the Black-Scholes option pricing theory is that a standard call option's value will increase with volatility (Hull 2000), recognized as a positive "vega". Such a move corresponds to the early rise in the dilution multiplier, with ESO holders gaining fractional ownership in the firm at the expense of long-term shareholders. Once volatility exceeds a critical value, however, the dilution multiplier decreases back toward 1.

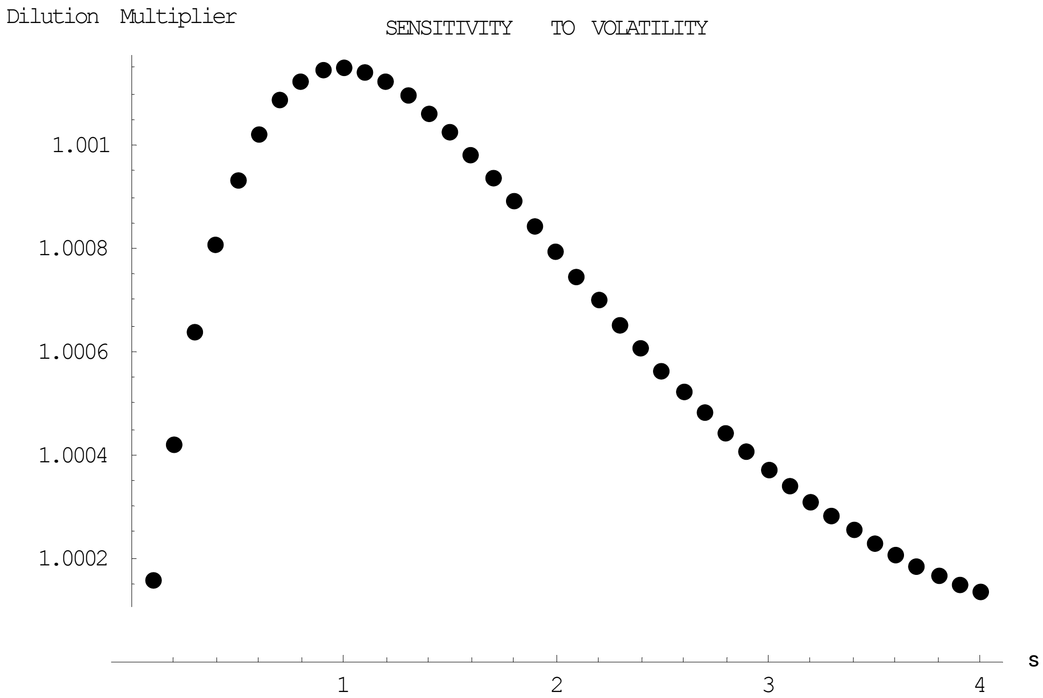

Figure 3

A less surprising result is found when the trend in dilution is investigated as a function of the number of tranches in an extended ESO award program. Figure 4 depicts this situation for the number of tranches, m, from 1 to 20, with exercise times $\tau_i = i$ years, exercise prices $K_i = 1.1S_0$, and other parameters as for Figure 2. The dilution multiplier increases almost linearly with the number of tranches in the program as long as the number of options per tranche is unchanged.

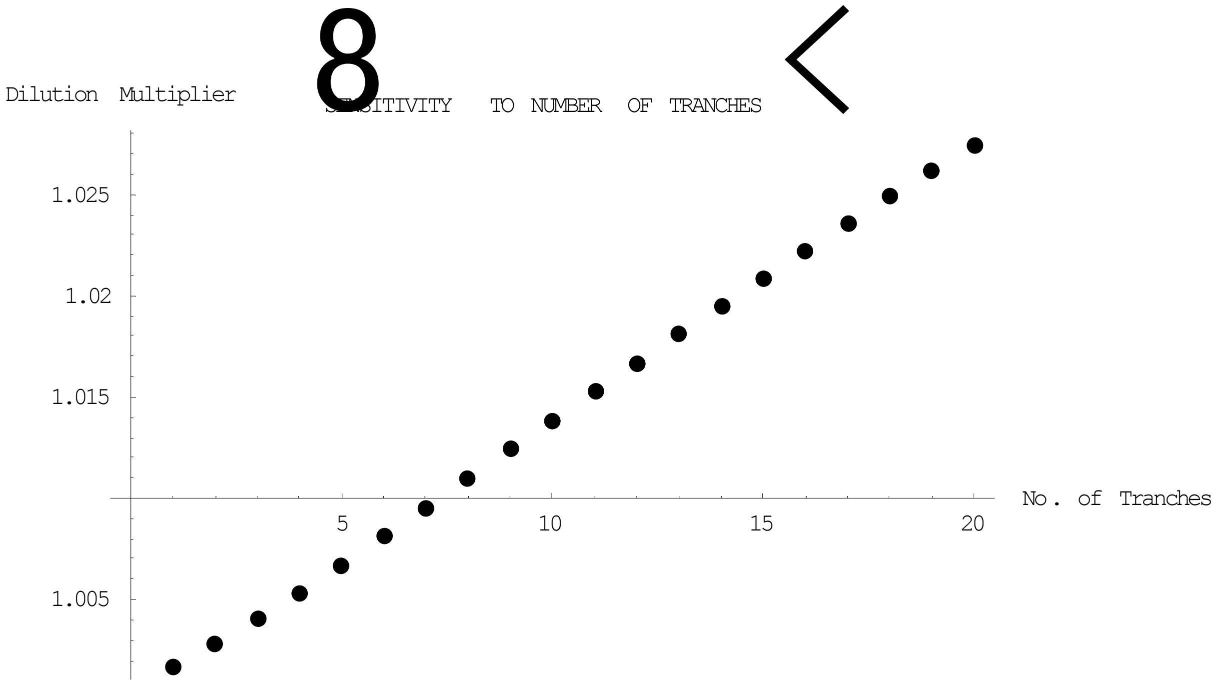

Figure 4

A consistent finding of our numerical simulations is that ESO programs have impacts on dilution that are generally small. In part, this results from the relatively small fraction of stock reserved for ESOs, but it is also a consequence of the stock repurchases that are made with exercise proceeds.

The equation (2.13) provides a closed-form expression for risk-neutral expected dilution that will be helpful to analysts in a number of ways, particularly in estimating future earnings per share and the future number of shares outstanding. As illustrated in the simulations, the sensitivity of dilution to stock price, exercise time or other model parameters can also be determined from the dilution formula.

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