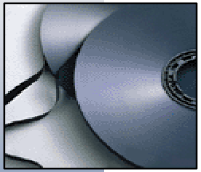
14th EAST ASIAN ACTUARIAL CONFERENCE

Bodily Injury Liability Loss Model & Credibility

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HEUNG KI, JUN
Hanwha Non-life Insurance CO.,LTD.

1. Introduction

2. The bodily Injury Liability Loss Model

3. Loss Model and Testing of Actual Data

4. Credibility of Bodily injury liability

5. Summary and Conclusion

1.1 Introduction and Purpose

- **In a view of actuarial science, liability has a special character of Long Tail Business because of factors such as compensation for damage based on the civil law and so on. Therefore, it is hard to estimate loss of liability.**

1.1 Introduction and Purpose

- **Estimating correct loss is not easy because it is a prospect of the future trend of loss. To solve this problem, actuarial method is used significantly.**
- **This topic is to devote to the development of insurance industry by presenting theoretical contents and practical models with Korean loss data of liability.**

1.2 Study of models

- **We will estimate character of distributions from character of samples expecting samples are similar to distributions. For example, we can regard sample mean, sample variance as an estimation of population mean and variance.**
- **We will find out how accurate estimations of loss are, and also how close they are to unknown parameters of future loss by modeling and finding frequency, severity, and other related variables.**

1.2 Study of models

- Through this process, we make it possible to forecast the operating achievements and the prospect of casualty covering Outstanding Reserve, Loss Ratio, Pricing Method.
- Thus, we applied and tested statistical models as Credibility to verify the consistency.

2.1 Observation period

- For the bodily injury liability, the period needs to be as long as possible to level off the change between years.
- we need a observation period more than 10 years in Japan, more than 5 years in Korea for the bodily liability.

Liability in Korea (FY2000~FY2006)

(unit: persons, 1,000,000won)

	Number of claims	Portion of claims	Portion of paid amount	Average paid amount
~ 1 year	753,283	83.2%	63.7%	498,051
1 year ~ 2 years	126,252	13.9%	25.3%	197,929
2 years ~ 3 years	17,652	1.9%	6.6%	51,257
3 years ~ 4 years	5,699	0.6%	3.2%	25,325
4 years ~ 5 years	1,788	0.2%	0.7%	5,336
5 years ~	842	0.1%	0.5%	4,060
Total	905,516	100.0%	100.0%	

Liability in Japan

(unit: persons, $)

	Number of claims	Portion of claims	Portion of paid amount	Average paid amount
~ 1 year	33	6.8%	3.6%	253,942
1 year ~ 2 years	100	20.6%	20.3%	467,379
2 years ~ 3 years	122	25.1%	24.4%	461,896
3 years ~ 4 years	81	16.7%	19.0%	541,123
4 years ~ 5 years	73	15.0%	12.9%	408,964
5 years ~ 10 years	72	14.8%	19.7%	630,158
10 years ~	5	1.0%	0.1%	32,790
Total	486	100.0%	100.0%	

2.2 Trend

- **We use trend factors to manipulate the past statistical data to reflect changes of loss that are regarded to increase until new rates are applied. The existing trend factors were made with the idea that trend of frequency or severity for previous years will continue in the predictable future.**

2.3 Sampling of statistical data

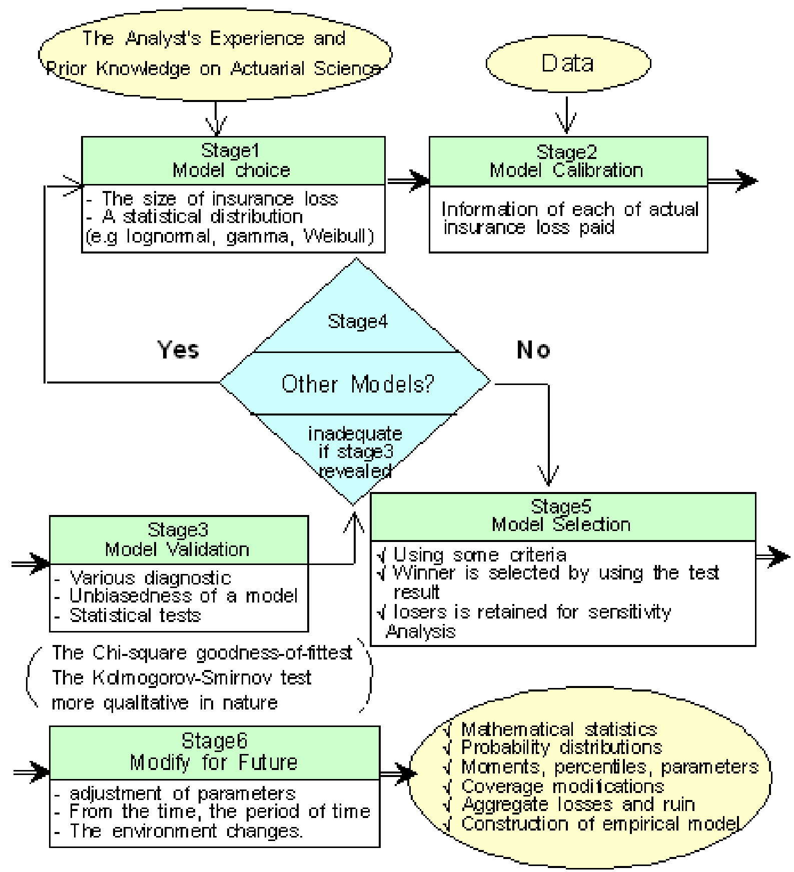
- **Loss Development Factor is used to correct the prediction error of loss which is reported but not paid yet and to predict IBNR reserve. Loss Development Factor is calculated under the decision that the trend of loss of past years will continue in future.**

[Table 2.1] Stratified Sampling Data

Incurred Year	2002	2003	2004	2005	2006
Victims	43,787	52,559	51,908	54,673	48,103

2.4 Loss Model Flow

- **Stage1 - Model choice**
- **Stage2 - Model Calibration**
- **Stage3 - Model Validation**
- **Stage4 - Other Models?**
- **Stage5 - Model Selection**
- **Stage6 - Modify for Future**



2.5 Loss model function

- **We can design a model of frequency and severity of accident by measuring X and x (X=x, X: random variable, x: value of X) with cumulative distribution function F(x), Survival function Sx(x), probability density function f(x), Hazard rate function h(x)**

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3.1 Testing of Actual Data

- The first table shows us actual number of victims and amount of paid benefit of bodily injury liability per year in Korea. The second table is about numbers of victims per grade of accident.

[Table 3.1] Number of victims and paid benefit per accident year

(unit: persons, ¥1,000,000)

Accident year	Victims	Paid benefit per development year				
		1 year	2 years	3years	4 years	5 years
2002	43,787	112,482	33,919	6,660	3,223	1,170
2003	52,559	129,919	39,493	7,514	3,678	
2004	51,908	122,233	34,345	8,264		
2005	54,673	134,733	49,163			
2006	48,103	134,431				

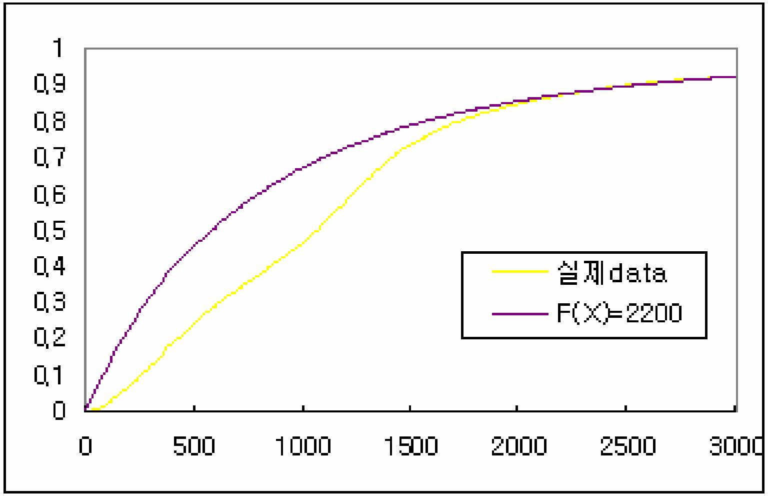
[Table 3.2] Number of victims per accident year

Injury grade	Accident rate			
	2004	2005	2006	2007 (estimation)
Death	0.03%	0.04%	0.03%	0.03%
Grade 1	0.05%	0.05%	0.05%	0.05%
Grade 2	0.04%	0.04%	0.03%	0.03%
Grade 3	0.03%	0.02%	0.02%	0.02%
Grade 4	0.04%	0.03%	0.03%	0.03%
Grade 5	0.09%	0.09%	0.08%	0.09%
Grade 6	0.05%	0.04%	0.04%	0.04%
Grade 7	0.10%	0.09%	0.09%	0.09%
Grade 8	2.35%	2.32%	2.26%	2.15%
Grade 9	5.17%	5.09%	5.28%	5.41%
Grade10	0.06%	0.06%	0.07%	0.07%
Grade11	0.12%	0.09%	0.08%	0.08%
Grade12	0.09%	0.09%	0.09%	0.09%
Grade13	0.24%	0.23%	0.25%	0.25%
Grade14	0.97%	1.07%	1.01%	0.98%
Total	9.43%	9.35%	9.43%	9.42%

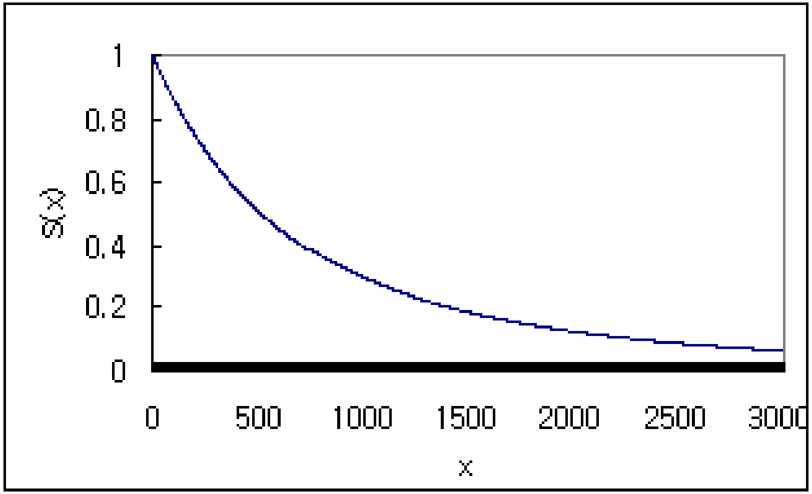
3.2 Probability model

$$F_1(x) = \begin{cases} 0 & , x < 0 \\ 1 - \left(\frac{2200}{x + 2200}\right)^3 & , x \geq 0 \end{cases}$$

$$S_1(x) = \left(\frac{2200}{x + 2200}\right)^3 \quad , x \geq 0$$



[figure 3.1] c.d.f

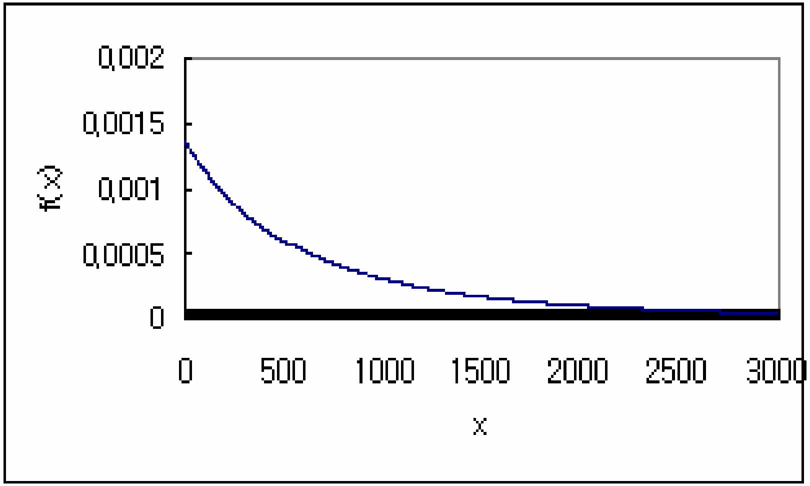


[figure 3.2] Survival function

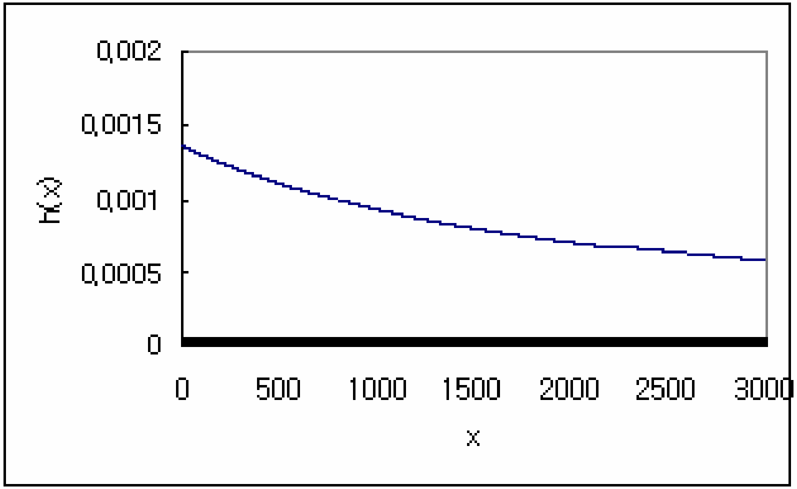
3.2 Probability model

$$f_1(x) = \frac{3(2200)^3}{(x+2200)^4}, \quad x > 0$$

$$h_1(x) = \frac{3}{x+2200}, \quad x > 0$$



[figure 3.3] Prob. Density function



[figure 3.4] Hazard rate function

3.2 Probability model

- **Distributions of the insured of bodily injury liability**

Discrete
(The random variable places probability only at 0,1,2,3,4(the support))

$$F_2(x)=\begin{cases}0 & ,x<0\\ 0.004 & ,0\le x<1\\ 0.009 & ,1\le x<2\\ 0.013 & ,2\le x<3\\ 0.016 & ,3\le x<4\\ 0.019 & ,0\le x<5\\ 0.029 & ,1\le x<6\\ 0.033 & ,2\le x<7\\ 0.042 & ,3\le x<8\\ 0.284 & ,0\le x<9\\ 0.816 & ,1\le x<10\\ 0.824 & ,2\le x<11\\ 0.833 & ,3\le x<12\\ 0.843 & ,2\le x<13\\ 0.889 & ,3\le x<14\\ 1 & ,x\ge 14\end{cases},\quad S_2(x)=\begin{cases}0 & ,x<0\\ 0.996 & ,0\le x<1\\ 0.991 & ,1\le x<2\\ 0.987 & ,2\le x<3\\ 0.984 & ,3\le x<4\\ 0.981 & ,0\le x<5\\ 0.971 & ,1\le x<6\\ 0.967 & ,2\le x<7\\ 0.958 & ,3\le x<8\\ 0.716 & ,0\le x<9\\ 0.284 & ,1\le x<10\\ 0.176 & ,2\le x<11\\ 0.167 & ,3\le x<12\\ 0.157 & ,2\le x<13\\ 0.111 & ,3\le x<14\\ 1 & ,x\ge 14\end{cases},\quad p_2(x)=\begin{cases}0.004 & ,x=0\\ 0.005 & ,x=1\\ 0.004 & ,x=2\\ 0.003 & ,x=3\\ 0.003 & ,x=4\\ 0.010 & ,x=5\\ 0.004 & ,x=6\\ 0.009 & ,x=7\\ 0.242 & ,x=8\\ 0.532 & ,x=9\\ 0.008 & ,x=10\\ 0.009 & ,x=11\\ 0.010 & ,x=12\\ 0.046 & ,x=13\\ 0.111 & ,x=14\end{cases}$$

3.3 Standard loss amount

- **The standard loss amount (including payable benefit in the future) is calculated to estimate the frequency of insurance accident and the average loss amount reasonably by considering the average paid benefit, inflation, and the average time of payment.**

3.3 Standard loss amount

- **Distributions of the insured of bodily injury liability.**

Process	Notes
Collect of base data	→ Total paid benefit and O/S (2002~2006)
↓	
Select data	→ Observation year: 2002~2006
↓	
Average Loss Amount per victim	
↓	
Safety Margin	→ O/S per injury grade =Max (Individual estimation, CLM)
↓	
Smoothness & Fitness	
↓	
Prediction of 2007	→ Correction factor X Inflation ratio

3.4 Loss development Method

- **We correct the Data on Run-off Triangle**
  : Accident year basis to Loss Development year.
- **That were separated into portfolio factor of Paid Loss Development and Inflation factor(λ,ɣ). Estimating the future loss in Development Year (≒Separation Method, Chain Ladder Method, etc)**

3.4 Loss development Method

[Table 3.3] Loss Amount of Accident years

(unit : ￥1,000,000)

Injury grade	2004	2005	2006	2007(estimation)
Death	22,615	26,005	26,767	25,129
Grade 1	31,621	28,512	29,793	29,975
Grade 2	7,446	7,003	5,579	6,676
Grade 3	5,606	4,622	4,094	4,774
Grade 4	6,011	6,254	5,699	5,988
Grade 5	8,917	8,817	8,672	8,802
Grade 6	3,570	3,382	3,130	3,361
Grade 7	5,870	5,604	5,777	5,751
Grade 8	36,150	36,626	32,537	35,104
Grade 9	56,012	56,492	53,999	55,501
Grade 10	618	601	549	589
Grade 11	1,014	855	750	873
Grade 12	855	835	845	845
Grade 13	942	1,881	3,324	2,049
Grade 14	2,410	2,797	4,655	3,287
Total	189,656	190,287	186,171	188,705

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4.1 Credibility of Bodily injury liability

- **We will see another model of 2 steps about loss of bodily injury liability. The analysis of frequency of benefit payment is based on Poisson distribution. The analysis of benefit amount is based on Exponential distribution. Let's assume that the number of benefit payments and individual benefit amount are independent. After seeing predictive distributions and posterior distributions, We will make models from distributions of the number of benefit payments and the benefit amount.**

4.2 Analysis of number of claims

- The formula has the form of the negative binomial density function.
- The mean of a negative binomial density function is $\frac{\alpha+mn}{\beta+m}$ which is equal to the estimation of Buhlmann's credibility about the number of benefit payment. The density function of a negative binomial distribution shows the predictive density function of the number of benefit payment during the (m+1) insurance period, if we are given $N_1 = n_1, N_2 = n_2, \cdots N_m = n_m$. $n = n_{m+1}$ has values of 0, 1, 2, ..
- This predictive density function provides (predictive) probability for each available benefit payment and more information than the predicted mean gives.

4.2 Analysis of number of claims

<u>The number of claims of bodily injury liability per year</u>

(unit : claim)

Year	2003	2004	2005	2006
Claims	51,518	50,909	49,552	48,775

- Applying actual data to a negative binomial distribution, the probability that claims are requested more than 50,000 in 2007 is

$$p[N_5 \geq 50{,}000 \mid N_1 = 51{,}518,\ N_2 = 50{,}909,\ N_3 = 49{,}552,\ N_4 = 48{,}775]$$

$$m = 4 \text{ and } m\bar{n} = \sum_{i=1}^{4} n_i = 51{,}518 + 50{,}909 + 49{,}552 + 48{,}775 = 200{,}754$$

- Using a negative binomial conditional probability, then we have

$$\sum_{n=6}^{\infty} \binom{\alpha + 200{,}754 + n - 1}{n} \left(\frac{1}{\beta+4+1}\right)^{n} \left(\frac{\beta+4}{\beta+4+1}\right)^{\alpha+200{,}754}$$

$$= \sum_{n=6}^{\infty} \binom{\alpha + n + 200{,}753}{n} \left(\frac{1}{\beta+5}\right)^{n} \left(\frac{\beta+4}{\beta+5}\right)^{\alpha+200{,}754}$$

$$= 1 - \sum_{n=0}^{5} \binom{\alpha + n + 200{,}753}{n} \left(\frac{1}{\beta+5}\right)^{n} \left(\frac{\beta+4}{\beta+5}\right)^{\alpha+200{,}754}$$

4.3 Analysis of benefit

- The forecasting density function of X reflects the ambiguousness of benefit amount as well as estimation of parameters. The function is

$$p(x \mid m',\ y',\ m\bar{n}, y) = \int_0^\infty p(x \mid \delta)\cdot f(\delta \mid m',\ y',\ m\bar{n}, y)\,d\delta$$

$$= C\int_0^\infty \left(\frac{e^{-x/\delta}}{\delta}\right)\left(\frac{e^{-(y+y)/\delta}}{\delta^{m+mn}}\right)d\delta$$

$$C = \frac{(y'+y)^{m+mn-1}}{\Gamma(m'+m\bar{n}-1)}$$

$$p(x \mid m',\ y',\ m\bar{n}, y) = C\int_0^\infty e^{-(x+y+y)/\delta}\,\delta^{-m-mn-1}\,d\delta$$

- This function is a density function belong to the family of Pareto.

4.3 Analysis of benefit

- Actually, the predictive density function of X (Incurred Loss) has a number of the Pareto family of density function.
- During the first four periods, 200,754 claims are requested and the total amount of payments is ¥401,508 million. The probability that a claim amount is more that the average benefit amount(¥2 million) is

$$p[x_{200,755} \geqq 200 \mid \sum_{i=1}^{200,754} X_i = 40,150,800]$$

- If the value of the parameter Δ is uncertain, comparatively a gentle exponential distribution is transformed to a thick-tailed Pareto distribution.

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- **We found out that the loss amount of bodily injury liability is not linear or discrete but exponentially distributed in probability models. The distribution function of the loss size is a constant function of sections.**
- **Credibility is a conditional density function. We also saw that the number of claim is negative binomially distributed and the benefit is Poissonly distributed. Both are under thick-tailed Pareto family.**

5. Summary and Conclusion

- Accident Year basis Incurred Loss Amount is not equivalent to that of Calendar-Policy year basis.
- According to circumstances, we have adopt Inflation factor.
- Any approach based on models should be treated according to the object of study. In actuarial statistics (science), many problems are about to make scientific models which will be used to predict insurance cost in the future.

5. Summary and Conclusion

- **Models are clear scientific description which is obtained from actuaries' experience and knowledge based on past data. Data is an index when actuaries decide unknown quantities called parameters and select a type of model. Models balance between the clearness and the fitness of available data.**
- **The clearness is measured by numbers of unknown parameters. The fitness for data is measured by the relation of discordance between models and data. Model selection is done based on the balance of two conditions, the fitness and the clearness.**