



# Building a Simplified Stochastic Asset Liability Model (ALM) for a Malaysian Participating Annuity Fund

14th EAST ASIAN ACTUARIAL CONFERENCE

Prepared by Seow Fan Chong, FSA

## Simplified Stochastic ALM – Malaysian Annuity Fund

- Lack of numerical demonstration of ALM for Asian insurance fund published
- Asian actuaries are increasingly required to use stochastic techniques in ALM
- Proprietary actuarial ALM software are complex and results difficult to verify
- Simplified stochastic ALM implemented in spreadsheet suffices for most Asian insurance companies

# Simplified Stochastic ALM – Malaysian Annuity Fund

## Outlines of Presentation

1. Overview of the Building Process
2. Key Features of a Typical Malaysian Participating Annuity Fund
3. Projection of Liability Cash-Flow
4. Simplified Stochastic Asset Models
5. Parameters Estimation and Assumptions
6. Simulation of Economic Series
7. Results of Stochastic ALM

# Simplified Stochastic ALM – Malaysian Annuity Fund

## Overview of the Building Process

1. Specify ALM Objectives
2. Project Liability Cashflow
3. Specify Stochastic Asset Models
4. Estimate Parameters of Asset Models
5. Simulate Economic Series
6. Project Asset Cashflow and combine with Liability Cashflow
7. Summarize Results as Balance Sheet and Profit & Loss Statement

# Simplified Stochastic ALM – Malaysian Annuity Fund

## Key Features of a Malaysian Annuity Product

1. Single Premium Deferred Participating Annuity
2. Entry Age: 19 to 54
3. Vesting Age: 55
4. Reversionary Bonus 2% pa
5. Annuity Guaranteed 10 years
6. Death before vesting: 10 years annuity certain
7. Surrender before vesting: 10 years annuity certain

# Simplified Stochastic ALM – Malaysian Annuity Fund

---

## Liability Assumptions

1. Expense – FY 1.5% SP; 5% Accrued Annuity
2. Commission – 3.5% SP
3. Lapse: 1-2 yr 20%; 3 to age 55 10%; 0% after
4. Mortality: 70% M83/88 before 55; a(90) set back 2 years after 55
5. Investment Income Tax 8%
6. Corporate Tax: 28% on transfer to shareholder
7. Reserving Method: Modified Net Premium
8. Reserving Basis: Interest 4.5%
9. Solvency Margin: 4% reserve + 0.2% Sum At Risk
10. Transfer to shareholder: 10% Cost of Bonus

# Simplified Stochastic ALM – Malaysian Annuity Fund

## Asset Models

1. Inflation – One Factor Vasicek Model
2. Real Interest – Two Factors Vasicek Model
3. Equity – Regime Switching Lognormal
4. Property - One Factor Vasicek Model

## Simplified Stochastic ALM – Malaysian Annuity Fund

### Inflation – One Factor Vasicek Model

$$\text{inf}_{t+1} = \text{inf}_t + \kappa_{\text{inf}} (\mu_{\text{inf}} - \text{inf}_t) \Delta t + \varepsilon_{\text{inf},t} \sigma_{\text{inf}} \sqrt{\Delta t}$$

$\mu_{\text{inf}}$  – long run mean inflation rate

$\kappa_{\text{inf}}$  – mean reversion speed

$\text{inf}_t$  – inflation rate at time t

$\sigma_{\text{inf}}$  – volatility of inflation rate

$\varepsilon_{\text{inf}}$  – standard normal error term

## Simplified Stochastic ALM – Malaysian Annuity Fund

### Real Interest – Two Factors Vasicek

$$\Delta s_t = \kappa_s (l_t - s_t) \Delta t + \sigma_s \varepsilon_{st}$$

$$\Delta l_t = \kappa_l (\mu_l - l_t) \Delta t + \sigma_l \varepsilon_{lt}$$

$$P(t, T) = A(t, T) e^{-r_t B_1(t, T) - l_t B_2(t, T)}$$

# Simplified Stochastic ALM – Malaysian Annuity Fund

## Real Interest – Two Factors Vasicek

$$B_1(t) = \left[ \frac{1 - e^{-\kappa_s t}}{\kappa_s} \right]$$

$$B_2(t) = \frac{\kappa_s}{\kappa_s - \kappa_l} \left[ \frac{1 - e^{-\kappa_l t}}{\kappa_l} - \frac{1 - e^{-\kappa_s t}}{\kappa_s} \right]$$

$$A(t) = (B_1(t) - t) \left[ \mu - \frac{\sigma_s^2}{2\kappa_s^2} \right] + B_2(t) \mu - \frac{\sigma_s^2 B_1(t)^2}{4\kappa_s}$$

$$+ \frac{\sigma_l^2}{2} \left[ \frac{t}{\kappa_l^2} - 2 \frac{(B_2(t) - B_1(t))}{\kappa_l^2} + \frac{1}{(\kappa_s - \kappa_l)^2} \frac{(1 - e^{-2\kappa_s t})}{2\kappa_s} - \frac{2\kappa_s}{\kappa_l(\kappa_s - \kappa_l)^2} \frac{(1 - e^{-(\kappa_s + \kappa_l)t})}{(\kappa_s + \kappa_l)} + \frac{\kappa_s^2}{\kappa_l^2(\kappa_s - \kappa_l)^2} \frac{(1 - e^{-2\kappa_l t})}{2\kappa_l} \right]$$

## Simplified Stochastic ALM – Malaysian Annuity Fund

### Equity – Regime Switching LogNormal

$$\log \frac{S_{t+1}}{S_t} \Big| \rho_t \sim N(\mu_{\rho t}, \sigma_{\rho t}^2)$$

$$\rho_{ij} = P[\rho_{t+1} = j \mid \rho_t = i] \quad i = 1, 2, j = 1, 2$$

$$\mathfrak{P} = \{\mu_1, \mu_2, \sigma_1, \sigma_2, \rho_{2,1}, \rho_{1,2}\}$$

# Simplified Stochastic ALM – Malaysian Annuity Fund

## Property – One Factor Vasicek Model

$$prop_{t+1} = prop_t + \kappa_{prop} (\mu_{prop} - prop_t) \Delta t + \varepsilon_{prop,t} \sigma_{prop} \sqrt{\Delta t}$$

$\mu_{prop}$  – long run mean property return

$\kappa_{prop}$  – mean reversion speed

$prop_t$  – property return at time t

$\sigma_{prop}$  – volatility of property return

$\varepsilon_{prop}$  – standard normal error term

# Simplified Stochastic ALM – Malaysian Annuity Fund

## Parameters Estimation - Inflation

$$\text{inf}_{t+1} = \alpha + \beta \text{inf}_t + \varepsilon'_{\text{inf},t}$$

$$\beta = (1 - \kappa_{\text{inf}} \Delta t)$$

$$\kappa_{\text{inf}} = \frac{1 - \beta}{\Delta t}$$

$$\alpha = \kappa_{\text{inf}} \mu_{\text{inf}} \Delta t = \frac{1 - \beta}{\Delta t} \mu_{\text{inf}} \Delta t$$

$$\mu_{\text{inf}} = \frac{\alpha}{1 - \beta}$$

# Simplified Stochastic ALM – Malaysian Annuity Fund

## Parameters Estimation - Inflation

<u><i>Time</i></u> <u><i>Period</i></u>	$\kappa_{inf}$	$\mu_{inf}$	$\sigma_{inf}$
1972-2006	0.49	3.9%	3.0%

## Simplified Stochastic ALM – Malaysian Annuity Fund

### Parameters Estimation – Interest Rates

$$\text{Stage 1: } l_{t+1} = \beta_1 + \beta_2 l_t + \varepsilon'_{2t}$$

$$\text{Stage 2: } \Delta s_{t+1} = \alpha_1 (\hat{l}_t - s_t) + \varepsilon'_{1t}$$

$$\beta_1 = \kappa_l \mu_l$$

$$\beta_2 = (1 - \kappa_l \Delta t)$$

$$\varepsilon'_{2t} = \sigma_l \varepsilon_{1t}$$

$$\alpha_1 = \kappa_s \Delta t$$

$$\varepsilon'_{1t} = \sigma_s \varepsilon_{st}$$

# Simplified Stochastic ALM – Malaysian Annuity Fund

## Parameters Estimation – Interest Rates

<i><u>Period</u></i>	$\kappa_s$	$\mu_l$	$\sigma_s$	$\kappa_l$	$\sigma_l$
96 – 06	1.48	3.04%	3.802%	12.01	4.43%

# Simplified Stochastic ALM – Malaysian Annuity Fund

## Parameters Estimation – Equity

<b><i>Period</i></b>	<b><math>\mu_1</math></b>	<b><math>\mu_2</math></b>	<b><math>\sigma_1</math></b>	<b><math>\sigma_2</math></b>	<b><math>\rho_{12}</math></b>	<b><math>\rho_{21}</math></b>
99–07	16.15%	2.99%	10.53%	23.92%	0.0109	0.0122

# Simplified Stochastic ALM – Malaysian Annuity Fund

## Parameters Estimation – Property

<u><i>Period</i></u>	$\kappa_{\text{prop}}$	$\mu_{\text{prop}}$	$\sigma_{\text{prop}}$
72-06	1.56	11.4%	2.7%

# Simplified Stochastic ALM – Malaysian Annuity Fund

## Parameters Estimation – Correlation

	<b>Inflation</b>	<b>Bond 1y</b>	<b>Bond 10y</b>	<b>Equity</b>	<b>Property</b>
<i><b>Inflation</b></i>	1	0.7753	0.6595	0.1119	0.0356
<i><b>Bond 1y</b></i>	0.7753	1	0.8714	(0.0615)	0.4113
<i><b>Bond 10y</b></i>	0.6595	0.8714	1	(0.1331)	0.2601
<i><b>Equity</b></i>	0.1119	(0.0615)	(0.1331)	1	0.0482
<i><b>Property</b></i>	0.0356	0.4113	0.2601	0.0482	1

# Simplified Stochastic ALM – Malaysian Annuity Fund

## Simulation of Economic Series

$$\text{inf}_{t+1} = \text{inf}_t + 0.49*(0.039 - \text{inf}_t) + 0.03\varepsilon_{\text{inf},t}$$

$$\Delta s_t = 1.48*(l_t - s_t) + 0.03802\varepsilon_{st}$$

$$\Delta l_t = 12.01*(0.0304 - l_t) + 0.0443\varepsilon_{lt}$$

# Simplified Stochastic ALM – Malaysian Annuity Fund

## Simulation of Economic Series

$$\log S_{t+1} = \log S_t + \mu_{pt} + \sigma_{pt} \varepsilon_{St}$$

Where

$$p_{1t} = 1, \mu_1 = 0.1615, \sigma_1 = 0.1053$$

$$p_{2t} = 2, \mu_2 = 0.0299, \sigma_2 = 0.2392$$

$$p_{11} = 0.9891, p_{21} = 0.0122,$$

$$p_{12} = 0.0109, p_{22} = 0.9878$$

$$\text{prop}_{t+1} = \text{prop}_t + 1.56 * ( 0.114 - \text{inf}_t ) + 0.027 \varepsilon_{\text{prop},t}$$

# Simplified Stochastic ALM – Malaysian Annuity Fund

## Selected Results of ALM

<b>Asset Class</b>	<b>Allocation Rate</b>
Cash	15.0%
MGS – 1 year	10.0%
MGS – 10 year	20.0%
PDS – 1 year	10.0%
PDS – 10 year	25.0%
Property	5.0%
Equity	15.0%

# Simplified Stochastic ALM – Malaysian Annuity Fund

## Selected Results of ALM

<b>Percentile</b>	<b>IRR</b>
5 percentile	Negative
25 percentile	3.00%
Mean	4.09%
75 percentile	4.58%
95 percentile	5.46%

## Selected Results of ALM

