Influenza Pandemic and an Actuarial Model That Takes Account Additional Mortality

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[1. Introduction]

- Summary of general information regarding current understanding of influenza pandemic and pandemic risk
- Extend traditional actuarial models including additional mortality caused by catastrophic type event
- A life insurance portfolio of N policies is examined to analyze its non-diversifiable risk
- Calculation of theoretical value of the variance of loss stochastic value and comparison with a simulation result
- <Note>

  The opinion presented here is solely based on the author's thoughts and would not represent the opinion by the company or the actuarial organization the author belongs to.

[2. Influenza Pandemic]

1) What is influenza pandemic?

2) What makes the influenza pandemic a big threat to our society?

3) Which influenza pandemic occurred in the 20th century?

4) How would the influenza spread among the people and becomes pandemic?

5) What is influenza virus? What is highly pathogenic and weakly pathogenic virus?

6) What is possible impact of pandemic?

- *Impact on the human lives*
- *Impact on the economy*

[2. Influenza Pandemic]

1. What is influenza pandemic?

- Influenza pandemic is a situation when a new influenza virus emerges and infects humans, starts to spread easily and sustainable among humans, and finally causes widespread epidemic among the world.
- This is a risk (pandemic risk) that might cause worldwide increase of death and social/economic confusion. Influenza Pandemic occurs periodically

[2. Influenza Pandemic]

2. What makes the influenza pandemic a big threat to our society?

- Influenza Pandemic occurs periodically
- Once occurred, it might be a world wide phenomenon with disastrous loss of human lives
- Highly pathogenic type, called H5N1 type virus, is expected
- Current development of transportation network has enabled the influenza much faster and among the continents
- Past experience may not be applied in the current situation.

[2. Influenza Pandemic]

3. Which influenza pandemic occurred in the 20th century?

1. **Spanish Influenza (1918-1919):** Total number of the patient counted 25-30% of the total population of the world. Case fatality rate (Death over the Infected) is more than 2.5%. In Japan, the number of patient is estimated as 23 million and the 380,000 people died. Virus type is H1N1.
2. **Asian Influenza (1957-1958):** Case fatality rate seems to be much lower than that of Spanish Influenza. Estimated excess number of death is more than 2 million people worldwide. Virus type is H2N2.
3. **Hong Kong Influenza (1968-1969):** The impact is even milder than that of Asian Influenza. Virus type is H3N2.

[2. Influenza Pandemic]

4. How would the influenza spread among the people and becomes pandemic?

- A situation is that there exist new (potential) types of avian influenza in birds (wild birds or poultry); however, there is no infection to human (WHO stage 1 and 2).
- A situation is that infection from birds to human is sometimes reported; however, the infection from human to human is not recognized (WHO stage 3, which is the current situation).
- A situation is that infection from human to human is recognized; however, the infection is still limited and under control (WHO stage 4 and 5)
- A situation is that infection from human to human is done in an efficient and sustained manner and thee influenza starts to spread (WHO stage 6, or pandemic)

[2. Influenza Pandemic]

5. What is influenza virus? What is highly pathogenic and weakly pathogenic virus?

- Three types, type A, B and C
- Hemagglutinin (HA) : 16 subtypes (H1 to H16)
- Neuraminidase (NA) : 9 subtypes (N1 to N9).
- Highly pathogenic virus:
  - H5 and H7 subtype is known as highly pathogenic
    - *Once virus with these subtypes infect human, it can attack human organ directly in the body and therefore, infected tends to show severe symptom and the fatality rate is higher.*
  - New influenza viruses found in Asian countries recently are H5N1 subtype, which foster the sense of crisis.
- Low pathogenic virus
  - Virus with other HA subtypes are known as low pathogenic.
  - All 20th century pandemics were caused by low pathogenic virus

[2. Influenza Pandemic]

6. What is possible impact of pandemic?

- The article by the Infectious Disease Surveillance Center of Japan provides the following list of impact

1) A huge number of patients and deaths
2) Mental and physical pain
3) Infection to the medical practicians
4) Excess demand to the medical institution and breakdown of medical services
5) Infection to the work staffs of social infrastructure (transportation, communication, police, electric, food, water , fire, etc.)
6) A breakdown of social function and public administration
7) Restriction on daily activities
8) Restriction on corporate activities
9) Collapse of the foundation of corporation, due to the domino effect
10) Decrease of population in production age
11) A huge economical loss.

[2. Influenza Pandemic]

6. What is possible impact of pandemic?

- The level of impact is affected by various factors. According to the above article,

1) Pathogenicity of the virus itself
2) Number of infected during an epidemic period
3) Frequency of severe complication
4) Distribution of patients' age
5) Speed of the pandemic spread
6) The degree of preparation
7) Effect of the governmental action
8) Social background and infrastructure (age distribution of the population, population density, sanitary condition in general
9) Level of the medical treatment.

[2. Influenza Pandemic]

6. What is possible impact of pandemic?

<Impact on Human lives>

- Ministry of Health, Labour and Welfare of Japan has estimated a number of deaths of the Japanese
- The model they used was originally developed by the Center of Disease Control (CDC) of the U.S.A
    - 25% of the total population of Japanese will be infected by the influenza virus.
    - The fatality rate is assumed to be 0.53% for moderate cases (similar to Asian influenza) and 2% for severe cases (similar to Spanish influenza)
    - The upper limits of deaths were estimated 170 thousand for severe cases and 640 thousands for severe cases.
    - The report remarks that effects (effectiveness) of factors such as intervention with new influenza vaccines or Antiviral drugs and sanitation conditions in Japan are not taken consideration in these estimates.

[2. Influenza Pandemic]

6. What is possible impact of pandemic?

<Impact on Economy>

- The reports prepared by the Congressional Budget Office of the U.S.A has concluded that
- The overall impact of a potential flu pandemic on gross domestic product to be
  - About 4.25% in a severe scenario
    - *one similar to Spanish influenza*
  - About 1% in the mild scenario.
    - *one similar to Asian or Hong Kong influenza.*

[3. Pandemic Risk]

- Definition:
  - From insures perspective…
  - pandemic risk is defined as the risk that occurs when influenza pandemic emerges and the excess claim may cause insurers' insolvency.
    - *Other than mortality risk is not considered in this presentation*
  - Pandemic risk may classified as a kind of catastrophe risk (or calamity risk) like earthquake risk, but has different risk characters

[3. Pandemic Risk]

- Comparison among mortality risk sources

Item	Pandemic	Earthquake	War/Terrorism	Underwriting risk in general
Area	May be global	Local	Assumed Local	—
Time horizon	Several weeks to several years	In a moment to several days	Several days to several decades	—
Cycle Length	Once in several decades	Once in several decades per an area	Unpredictable	Death occurs continuously
Impact on Human/Property	Human	Both Human/Property	Both Human/Property	Human
Impact by age	May take various patterns	Uniform among ages	May be different	As mortality table
Distribution of loss	Distribution with a long tail	Distribution with a long tail	Unknown	Normal distribution (Law of large numbers)
Quantitative evaluation of loss	Difficult	Some experience in PC insurers	Difficult	Sufficient experience in life insurers
Risk diversification	Not diversifiable	Not diversifiable	Not diversifiable	Diversifiable
Approach to risk	Contingency reserve, Surplus, Reinsurance, Securitization	Contingency (Catastrophe) reserve, Surplus, Reinsurance, Securitization	Exclusion clause	Law of large numbers, Risk selection, Contingency reserve, Surplus, Reinsurance, Securitization

[3. Pandemic Risk]

- Comparison between Pandemic risk and earthquake risk

<table>
<tr><th></th><th>Similarity</th><th>Difference</th></tr>
<tr><td>Pandemic Risk</td><td rowspan="2">- Catastrophe (Calamity) risk
-Occurs periodically in every several decades
- Long-tail distribution
- Non-diversifiable risk
- Exclusion clause might be applied
- Need government level preparation to ease losses
-(Currently in Japan) Contingency reserve I might be used</td><td>- Might be global (worldwide) in a long term (might be local in a short-term)
- The affect may continue for more than several years
- Different impact on humans by their ages
- Mainly impact on human lives
- Early stage counter-action might be critical to prevent the pandemic
- Quantitative risk evaluation might be difficult (Model risk and parameter risk might be significant)</td></tr>
<tr><td>Earthquake Risk</td><td>- Limited to local
- Occurs in a very short period (several days)
- Impact on all humans in a same degree
- Impact on both human lives and property
- Some experience of quantitative risk evaluation</td></tr>
</table>

[3. Pandemic Risk]

- Characteristics of influenza pandemic
  - Global vs. local
  - Pattern of impact among ages
    1. *Severe on childhood and elderly*
    2. *Severe on adulthood*
    3. *Impact is uniform*
  - governmental/social reaction to the pandemic
  - Risk quantification

[3. Pandemic Risk]

- Approaches to mortality risk management
  - Law of large numbers
  - Risk selection at policy issue
  - Contingency Reserve
  - Surplus accumulation
  - (Non-proportional) Reinsurance
  - Securitization

[3. Pandemic Risk]

- Pandemic risk models
  - CDC model
  - SIR models
  - Agency based models
- Risk quantification
  - Stochastic simulation models
    - *Can determine VaR and TVaR*
    - *Incorporate characteristics of influenza pandemic*
    - *Some reinsurers and risk expert companies start to provide commercial pandemic risk simulation models that incorporate SIR model and agency based model approaches.*
  - This paper's approach
    - *Extension of life contingencies*
    - *Can calculate theoretical variance and standard deviation*
    - *Simple and conventional, supplement simulation approach*

[4. A Review of Traditional Life Contingencies]

- (Reference) Actuarial Mathematics (SOA)
- Notations:

$$T = T(x) : stochastic\ \mathrm{var}iables\ that\ means\ future\ lifetime\ of\ a\ life\ of\ age(x)$$

$$F_T(t) = \Pr[T \le t] : Cummulative\ probability\ distribution\ funcition\ of\ T$$

$$f_T(t) = \frac{\partial F_T(t)}{\partial t} = {}_{\mathrm{t}}\mathrm{p}_{\mathrm{x}}\,\mu(x+t) : \Pr obability\ density\ function\ of\ T$$

$$\mu(x+t) = -\frac{1}{F_T(t)}\frac{\partial F_T(t)}{\partial t} = \frac{f_T(t)}{F_T(t)} : Force\ of\ mortality$$

- We assume that a set of continuous mortality is provided a priori in some way

[4. A Review of Traditional Life Contingencies]

- If we are given a mortality table $\{q_x\}$ (x=0,1,2,・・・), which is a discrete mortality information, we will convert mortality rate of each age into force of mortality by assuming a uniform distribution of death between age x+t and x+t+1.
- For x and t as integer and for $0 \leqq s < 1$, we assume that,

$$\mu(x+t+s) = \frac{q_{x+t}}{1 - sq_{x+t}}$$

$${}_s p_{x+t} = 1 - sq_{x+t}$$

[4. A Review of Traditional Life Contingencies]

- Consider continuous premium pay, immediate benefit pay whole life policy,

  $Z = e^{-\delta T}$: *Stochastic* var*iable that represents*

  present value of future benefit payment

- $$\overline{A}_x = E[Z] = \int_0^{\omega - x} e^{-\delta t} {}_t p_x \mu(x+t)dt = \frac{i}{\delta} \sum_{t=0}^{w-x} e^{-\delta(t+1)} {}_{t|}q_x = \frac{i}{\delta} A_x$$

  $$Var[Z] = {}^{2\delta}\overline{A}_x - (\overline{A}_x)^2$$

  $$E[e^{-\delta T} - \overline{P}_x \times \overline{a}_{\overline{T|}}] = 0$$

- (Note) Symbols shown on the upper left of a notation represents a force on interest used for discounting, for example,

$$ {}^{2\delta}\overline{A}_x = \int_0^{w-x} e^{-2\delta t} {}_t p_x \mu(x+t)dt$$

[4. A Review of Traditional Life Contingencies]

- Let L represents present value of future loss of insurance company for a policy at policy issue date,

  $$L = e^{-\delta T} - \overline{P}_x \times \overline{a}_{\overline{T|}}$$

- $$L = \frac{(\delta + \overline{P}_x)Z - \overline{P}_x}{\delta}$$

  $$E[L] = 0 \ \ (Equivalence\ principle)$$

  $$Var[L] = (1 + \frac{\overline{P}_x}{\delta})^2 Var[Z]$$

- Let ${}^{Port}L$ represents the future loss stochastic variable for the insurance portfolio that consists n policies.

  $${}^{Port}L = L_1 + L_2 + \Lambda\ L_n$$

  $$Var[{}^{Port}L] = nVar[L] + \sum_{k \neq l} Cov[L_k, L_l] = n(1 + \frac{\overline{P}_x}{\delta})^2 Var[Z]$$

[5. Extended Risk Models that Considers Additional Mortality]

- Assume a kind of catastrophic event such influenza pandemic or earthquake, and extend the standard life contingency model including these event.
- In addition to usual mortality, we assume that when a catastrophic event occurs, **1-R of the total population dies while the other R survives** on that moment (R might be treated either as a stochastic variable or as a constant value, depending on the model).

[5. Extended Risk Models that Considers Additional Mortality]

- <Model A>
  - The event occurs once in 1/ λ years in average.
  - The event may occur not once.
  - R is constant (=r)
  - The event occurs with a constant probability in the future. In other words, the interval stochastic variable, U_1, U_2... between two the events follows an exponential distribution with average 1/ λ .

$$F_U(u) = 1 - \exp(-\lambda u)$$

where U_1, U_2, ... are independent each other.

  - It is well known that, if there are K catastrophic event occurred in time 0 to t, K follows Poisson distribution with average λ t;

$$\Pr(K = k) = e^{-\lambda t} \frac{(\lambda t)^k}{k!}$$

[5. Extended Risk Models that Considers Additional Mortality]

- <Model B>
  - A catastrophic event occurs only once.
  - U that represents the time of stochastic event follows exponential distribution with average $1/\lambda$

$$F_U(u) = 1 - \exp(-\lambda u)$$

  - R is a stochastic variable that takes a value between 0 and 1, and E[R]=r, Var[R]= σ^2.

[6. Expression of Some Actuarial Notations]

- For Model A, we have the following formula
- Note the star signs $*(*^{A}, *^{B})$ on the upper right implies that additional mortality by the catastrophic event is included
- Cumulative probability distribution function of T^{*A}

$$
\begin{aligned}
F_{T^*}(t) &= 1 - \sum_{k=0}^{\infty} r^k \times {}_t p_x \times e^{-\lambda t} \frac{(\lambda t)^k}{k!} \\
&= 1 - {}_t p_x \times e^{-\lambda t} \sum_{k=0}^{\infty} \frac{(r\lambda t)^k}{k!} \\
&= 1 - {}_t p_x \times e^{-\lambda(1-r)t}
\end{aligned}
$$

[6. Expression of Some Actuarial Notations]

- Force of mortality of T^{*A}

$$\mu^{*A}(x+t) = \mu(x+t) + \lambda(1-r)$$

- Some comment
  - The force of mortality is given by an assumed mortality (1st term) plus additional constant mortality (2nd term)
  - Additional constant mortality may come from, for example,
    - *Continuous exposure to additional risk of an insured who has hazard occupation such as a pilot*
    - *Mortality increase by an influenza pandemic risk*
  - We cannot tell the nature of the risk by just looking at the force of mortality
  - Later we will examine the joint cumulative probability distribution function, where we can distinguish non-diversifiable risk from diversifiable risk

[6. Expression of Some Actuarial Notations]

- Probability density function

$$f_{T^*}(t) = {}_t p_x \times \mu(x+t) \times e^{-\lambda(1-r)t} + \lambda(1-r) \times {}_t p_x \times e^{-\lambda(1-r)t}$$

- Single premium

$$\overline{A}_x^* = E[Z^*]$$

$$= \int_0^{\omega-x} e^{-\delta t} \times f_{T^*}(t)dt$$

$$= \int_0^{\varpi-x} e^{-\delta t} \times ({}_t p_x \times \mu(x+t) \times e^{-\lambda(1-r)t} + \lambda(1-r) \times {}_t p_x \times e^{-\lambda(1-r)t})dt$$

$$= \int_0^{w-x} e^{-(\delta+\lambda(1-r))t} \times {}_t p_x \times \mu(x+t)dt + \lambda(1-r)\int_0^{w-x} e^{-(\delta+\lambda(1-r))t} \times {}_t p_x dt$$

[6. Expression of Some Actuarial Notations]

- Using following notions, we have a closed expression.

$$ {}^{\delta+\lambda(1-r)}\overline{A}_x = \int_0^{w-x} e^{-(\delta+\lambda(1-r))t} \times {}_t p_x \times \mu(x+t)dt $$

$$ {}^{\delta+\lambda(1-r)}\overline{a}_x = \int_0^{w-x} e^{-(\delta+\lambda(1-r))t} \times {}_t p_x dt $$

- $$ \begin{aligned} \overline{A}_x^* &= {}^{\delta+\lambda(1-r)}\overline{A}_x + \lambda(1-r) \times {}^{\delta+\lambda(1-r)}\overline{a}_x \\ &= 1 - \delta \times {}^{\delta+\lambda(1-r)}\overline{a}_x \\ &= \frac{1}{\delta+\lambda(1-r)}\left(\lambda(1-r) + \delta \times {}^{\delta+\lambda(1-r)}\overline{A}_x\right) \end{aligned} $$

$$ \overline{a}_x^* = {}^{\delta+\lambda(1-r)}\overline{a}_x $$

[6. Expression of Some Actuarial Notations]

- Variance of benefit payment present value

$$Var[Z^*] = \frac{1}{2\delta+\lambda(1-r)}\left(\lambda(1-r)+2\delta\times{}^{2\delta+\lambda(1-r)}\overline{A}_x\right) - \left(\frac{1}{\delta+\lambda(1-r)}\left(\lambda(1-r)+\delta\times{}^{\delta+\lambda(1-r)}\overline{A}_x\right)\right)^2$$

- Variance of future loss present value

$$L^* = \frac{(\delta+\overline{P}_x^*)Z^* - \overline{P}_x^*}{\delta}$$

$$Var[L^*] = (1+\frac{\overline{P}_x^*}{\delta})^2 \times Var[Z^*]$$

[6. Expression of Some Actuarial Notations]

- For Model B, similar argument follows that,
- Cumulative probability distribution function of T^{*B}, (first we assume R=r(constant))

$$F_{T^*}(t) = (1 - {}_tp_x) \times e^{-\lambda t} + (1 - r \times {}_tp_x) \times (1 - e^{-\lambda t})$$

$$= 1 - {}_tp_x \times (r + (1-r)e^{-\lambda t})$$

- This expression is valid for all R with E[R]=r
  - i.e. distribution of R is not necessary in the above expression

[6. Expression of Some Actuarial Notations]

- Stochastic probability density function

$$f_{T^*}(t) = {}_tp_x \times \mu_{x+t} \times (r + (1-r)e^{-\lambda t}) + {}_tp_x \times (1-r)\lambda e^{-\lambda t}$$

- Single premium

$$\overline{A}_x^* = r\overline{A}_x + (1-r)\,{}^{(\delta+\lambda)}\overline{A}_x + (1-r) \times \lambda\,{}^{(\delta+\lambda)}\overline{a}_x$$

$$= r\overline{A}_x + (1-r) \times \left(\frac{\lambda}{\delta+\lambda} + \frac{\delta}{\delta+\lambda}\,{}^{(\delta+\lambda)}\overline{A}_x\right)$$

$$\overline{a}_x^* = r \times \overline{a}_x + (1-r) \times {}^{(\delta+\lambda)}\overline{a}_x$$

[6. Expression of Some Actuarial Notations]

- Variance of benefit payment present value

$$Var[Z^*] = r\,{}^{2\delta}\overline{A}_x + (1-r)\times\left(\frac{\lambda}{2\delta+\lambda} + \frac{2\delta}{2\delta+\lambda}\,{}^{(2\delta+\lambda)}\overline{A}_x\right) - \left(r\,\overline{A}_x + (1-r)\times\left(\frac{\lambda}{\delta+\lambda} + \frac{\delta}{\delta+\lambda}\,{}^{(\delta+\lambda)}\overline{A}_x\right)\right)^2$$

- Variance of future loss present value

$$L^{*B} = \frac{(\delta + \overline{P}_x^{*B})Z^{*B} - \overline{P}_x^{*B}}{\delta}$$

$$Var[L^{*B}] = \left(1 + \frac{\overline{P}_x^{*B}}{\delta}\right)^2 \times Var[Z^{*B}]$$

[6. Expression of Some Actuarial Notations]

- An example
  - Mortality table: 2007 Japan Standard Table (Mail)
  - Assumed Interest: 2%
  - λ =0.02, r=0.98

Net Premium Rate

Age	30	50	70
Std Model	0.01269	0.02522	0.06383
Model A	0.01296	0.02548	0.06411
Model B	0.01289	0.02543	0.06408

Relative value (Std model =100%)

Age	30	50	70
Std Model	100.00%	100.00%	100.00%
Model A	102.18%	101.06%	100.43%
Model B	101.64%	100.86%	100.38%

Single Premium

Age	30	50	70
Std Model	0.39047	0.55061	0.76323
Model A	0.39562	0.55331	0.76400
Model B	0.39434	0.55281	0.76392

Relative value (Std model =100%)

Age	30	50	70
Std Model	100.00%	100.00%	100.00%
Model A	101.32%	100.49%	100.10%
Model B	100.99%	100.40%	100.09%

[6. Expression of Some Actuarial Notations]

- Simulated Result
  - For x=30, male, the theoretical calculation is compared with simulated result

	Standard Model			Model A			Model B		
	Formula(1)	Sim(2)	(2)/(1)	Formula(1)	Sim(2)	(2)/(1)	Formula(1)	Sim(2)	(2)/(1)
Single Premium	0.39047	0.39046	99.998%	0.39562	0.39561	99.998%	0.39434	0.39434	99.999%
Net Premium Rate	0.01269	0.01269	99.997%	0.01296	0.01296	99.997%	0.01289	0.01289	99.998%

[7. Loss of a Life Insurance Portfolio]

- An life insurance portfolio of n whole life policies
- Let
  - L^*_k be present value of loss for kth insured (k=1,2,･･･n)
  - Sum of L^*_k be $^{Port}L^*$

$$L^*_k = \frac{(\delta + \overline{P}^*_x) Z^*_k - \overline{P}^*_x}{\delta}$$

$$Var[L^*_k] = (1 + \frac{\overline{P}^*_x}{\delta})^2 Var[Z^*_k]$$

$$Cov[L^*_k, L^*_l] = (1 + \frac{\overline{P}^*_x}{\delta})^2 Cov[Z^*_k, Z^*_l] \quad (k \neq l)$$

$$^{Port}L^* = L^*_1 + L^*_2 + \Lambda \; L^*_n$$

$$Var[^{Port}L^*] = nVar[L^*] + \sum_{k \neq l} Cov[L^*_k, L^*_l]$$

[7. Loss of a Life Insurance Portfolio]

- For either Model A or Model B,

$$L^*_k \ and \ L^*_l \ (k \neq l)$$

are not independent. In other words, following expression is not zero.

$$Cov[L^*_k, L^*_l] = (1 + \frac{\overline{P}^*_x}{\delta})^2 Cov[Z^*_k, Z^*_l] \ \ (k \neq l)$$

- For the k^th^ and the l^th^ insured, the death event of two insured occurs randomly based on the mortality table and the catastrophic event; however, the catastrophic event affects both insured simultaneously and in the same impact and therefore, there is some correlation between the two mortality.

[7. Loss of a Life Insurance Portfolio]

- To calculate Cov[L^*_k,L^*_l] or Cov[Z^*_k,Z^*_l] for Model A,
- First, we calculate the joint probability density function of lifetime variable, T^*_1 and T^*_2 , of the two different insured,

$$F_{T_1^*,T_2^*}(t_1,t_2) = \Pr[T_1^* \le t_1, T_2^* \le t_2]$$

- We have the following expression (proof follows),

$$F_{T_1^*,T_2^*}(t_1,t_2) = (1 - {}_{t_1}p_x e^{-\lambda(1-r)t_1})(1 - {}_{t_2}p_x e^{-\lambda(1-r)t_2}) + {}_{t_1}p_x\, {}_{t_2}p_x e^{-\lambda(1-r)(t_1+t_2)}(e^{\lambda Min(t_1,t_2)(1-r)^2} - 1)$$

  - The second term implies that the there is some correlation between the two variables
  - Note that the above expression is valid when the age of the second insured is (y)

[7. Loss of a Life Insurance Portfolio]

- Discrete version of the joint probability density function is determined for m_1=1,2,…、m_2=1,2,…, ;

$$f_{T_1^*,T_2^*}(m_1,m_2) = \Pr(m_1 - 1 \le T_1^* < m_1, m_2 - 1 \le T_2^* < m_2)$$

$$= F_{T_1^*,T_2^*}(m_1,m_2) + F_{T_1^*,T_2^*}(m_1 - 1, m_2 - 1) - F_{T_1^*,T_2^*}(m_1 - 1, m_2) - F_{T_1^*,T_2^*}(m_1, m_2 - 1)$$

- Assuming the constant number of death assumption, we have

$$Cov[Z^*_1, Z^*_2] = (\frac{i}{\delta})^2 \sum_{m_1=1}^{\infty} \sum_{m_2=1}^{\infty} e^{-\delta(m_1+m_2)} f_{T_1^*,T_2^*}(m_1,m_2)$$

[7. Loss of a Life Insurance Portfolio]

- (Proof) First let $t_1 < t_2$,

$$F_{T_1^*,T_2^*}(t_1,t_2) = \sum_{u=0}^{\infty}\sum_{v=0}^{\infty}(1-r^u\, {}_{t_1}p_x)(1-r^{u+v}\, {}_{t_2}p_x)\times e^{-\lambda t_1}\frac{(\lambda t_1)^u}{u!}\times e^{-\lambda(t_2-t_1)}\frac{(\lambda(t_2-t_1))^v}{v!}$$

$$= 1 - {}_{t_1}p_x\sum_{u=0}^{\infty}e^{-\lambda t_1}\frac{(\lambda r t_1)^u}{u!} - {}_{t_2}p_x\sum_{u=0}^{\infty}e^{-\lambda t_1}\frac{(\lambda r t_1)^u}{u!}\sum_{v=0}^{\infty}e^{-\lambda(t_2-t_1)}\frac{(\lambda r(t_2-t_1))^v}{v!}$$

$$+ {}_{t_1}p_x\, {}_{t_2}p_x\sum_{u=0}^{\infty}e^{-\lambda t_1}\frac{(\lambda r^2 t_1)^u}{u!}\sum_{v=0}^{\infty}e^{-\lambda(t_2-t_1)}\frac{(\lambda r(t_2-t_1))^v}{v!}$$

$$= 1 - {}_{t_1}p_x e^{-\lambda(1-r)t_1} - {}_{t_2}p_x e^{-\lambda(1-r)t_2} + {}_{t_1}p_x\, {}_{t_2}p_x \exp(-\lambda t_1 + \lambda r^2 t_1 - \lambda(t_2-t_1) + \lambda r(t_2-t_1))$$

$$= (1 - {}_{t_1}p_x e^{-\lambda(1-r)t_1})(1 - {}_{t_2}p_x e^{-\lambda(1-r)t_2}) + {}_{t_1}p_x\, {}_{t_2}p_x e^{-\lambda(1-r)(t_1+t_2)}(e^{t_1\lambda(1-r)^2} - 1)$$

- Consider $t_1 < t_2$ case and symmetric property, we have the result

[7. Loss of a Life Insurance Portfolio]

- Once we have covariance,

$$Var[{}^{Port}L^*] = (1+\frac{\overline{P}_x^*}{\delta})^2(nVar[Z^*]+n(n-1)Cov[Z^*_1, Z^*_2])$$

$$Var[\frac{{}^{Port}L^*}{n}] = (1+\frac{\overline{P}_x^*}{\delta})^2(\frac{1}{n}Var[Z^*]+(1-\frac{1}{n})Cov[Z^*_1, Z^*_2])$$

- When n goes to infinity, the right term do not converge to zero. In other words, this insurance portfolio has non-diversifiable risk.

$$Var[\frac{{}^{Port}L^*}{n}] \approx (1+\frac{\overline{P}_x^*}{\delta})^2 Cov[Z^*_1, Z^*_2]$$

[7. Loss of a Life Insurance Portfolio]

- Also for Model B, we have the similar argument.
- Let , E[R]=r , and Var[R]= σ^2 ,

$$F_{T_1^*,T_2^*}(t_1,t_2)$$

$$= (1-r\,{}_{t_1}p_x)(1-r\,{}_{t_2}p_x) - (1-r)({}_{t_1}p_x e^{-\lambda t_1} + {}_{t_2}p_x e^{-\lambda t_2}) + (1-r)\,{}_{t_1}p_x\,{}_{t_2}p_x(re^{-\lambda Min(t_1,t_2)} + e^{-\lambda Max(t_1,t_2)})$$

$$+\sigma^2\,{}_{t_1}p_x\,{}_{t_2}p_x(1-e^{-\lambda Min(t_1,t_2)})$$

- Joint cumulative probability distribution function contains only information of expected value and variance of R.
- Also, when σ^2=0, the 4^{th} term becomes zero.

[7. Loss of a Life Insurance Portfolio]

- (Proof) First, let $t_1 < t_2$,

$$
\begin{aligned}
&F_{T_1^*,T_2^*}(t_1,t_2) \\
&= \int_0^1 g_R(s)((1-s\,{}_{t_1}p_x)(1-s\,{}_{t_2}p_x)(1-e^{-\lambda t_1}) + (1-{}_{t_1}p_x)(1-s\,{}_{t_2}p_x)(e^{-\lambda t_1}-e^{-\lambda t_2}) + (1-{}_{t_1}p_x)(1-{}_{t_2}p_x)(e^{-\lambda t_2})) \\
&= \int_0^1 g_R(s)((1-s\,{}_{t_1}p_x)(1-s\,{}_{t_2}p_x) - (1-s)({}_{t_1}p_x e^{-\lambda t_1} + {}_{t_2}p_x e^{-\lambda t_2}) + (1-s)\,{}_{t_1}p_x\,{}_{t_2}p_x(se^{-\lambda t_1}+e^{-\lambda t_2})) \\
&= (1-r\,{}_{t_1}p_x)(1-r\,{}_{t_2}p_x) + \sigma^2\,{}_{t_1}p_x\,{}_{t_2}p_x - (1-r)({}_{t_1}p_x e^{-\lambda t_1} + {}_{t_2}p_x e^{-\lambda t_2}) \\
&\qquad + {}_{t_1}p_x\,{}_{t_2}p_x((r-r^2-\sigma^2)e^{-\lambda t_1} + (1-r)e^{-\lambda t_2}) \\
&= (1-r\,{}_{t_1}p_x)(1-r\,{}_{t_2}p_x) - (1-r)({}_{t_1}p_x e^{-\lambda t_1} + {}_{t_2}p_x e^{-\lambda t_2}) + (1-r)\,{}_{t_1}p_x\,{}_{t_2}p_x(re^{-\lambda t_1}+e^{-\lambda t_2}) \\
&\qquad + \sigma^2\,{}_{t_1}p_x\,{}_{t_2}p_x(1-e^{-\lambda t_1})
\end{aligned}
$$

- Consider $t_1 < t_2$ case and symmetric property, we have the result
- Above formula is valid if the age of the 2nd insured is (y)

[8. Simulation Example]

- Using the assumption below, the loss of the insurance portfolio is simulated and compared with theoretical values.
- Simulation assumptions:
  - Mortality table: 2007 Japan Standard Table (Male), (Constant number of death is assumed in the fraction of years)
  - Assumed interest: 2%
  - A continuous premium whole life insurance portfolio of x=30, male, of n=21,000
  - Withdrawal other than death is not assumed.
  - Insurance amount: 10 million yen per policy, Simulation: 10,800 times
  - Model Parameters:

SIM1	Standard model	λ =0.00, R=r=1
SIM2	Model A	λ =0.02, R=r=0.98(Constant)
SIM3	Model B1	λ =0.02, R=r=0.98(Constant)
SIM4	Model B2	λ =0.02, $g_R(s)=49s^{48}$,E[R]=0.98

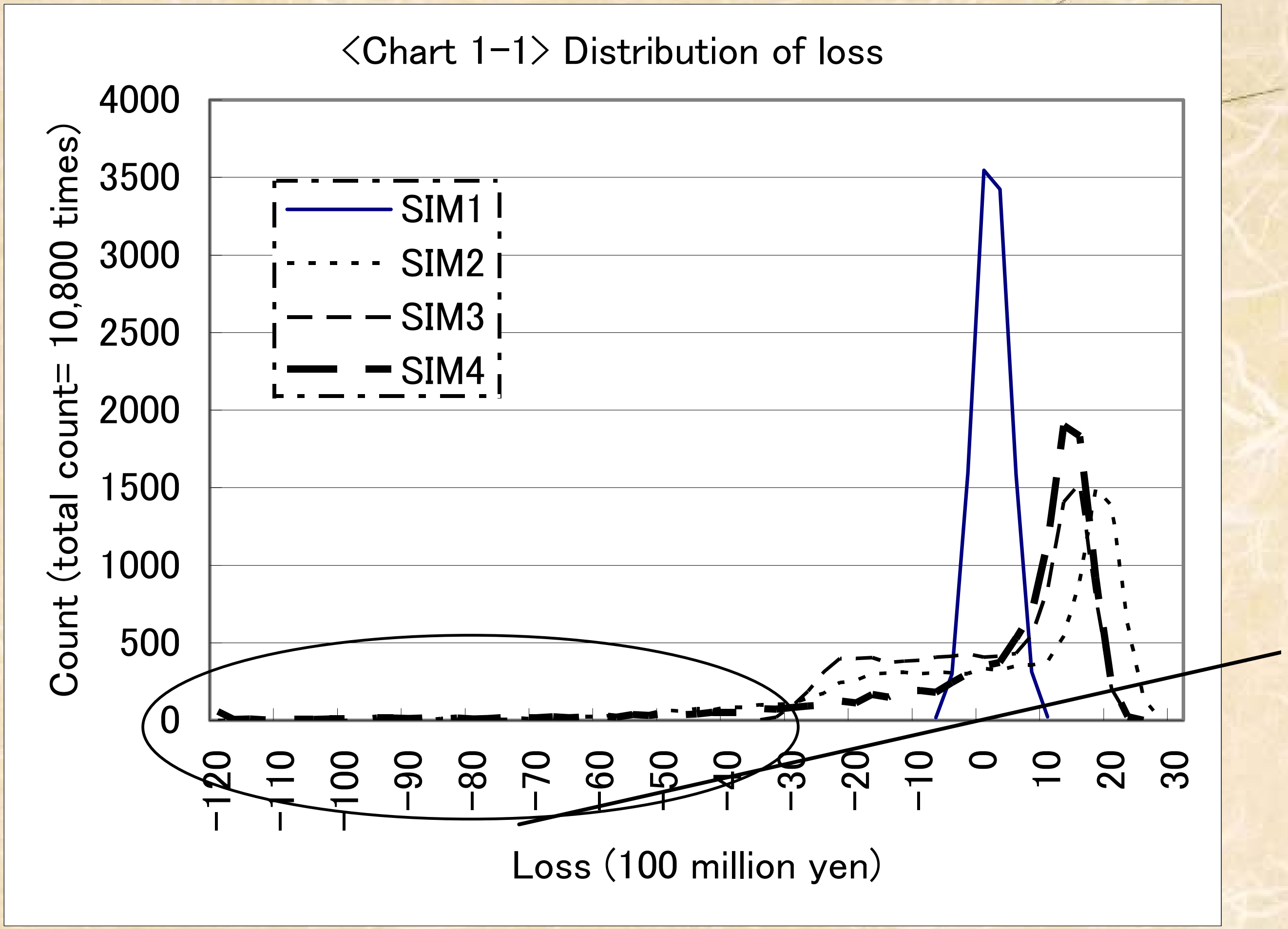
[8. Simulation Example]

- Simulated Result

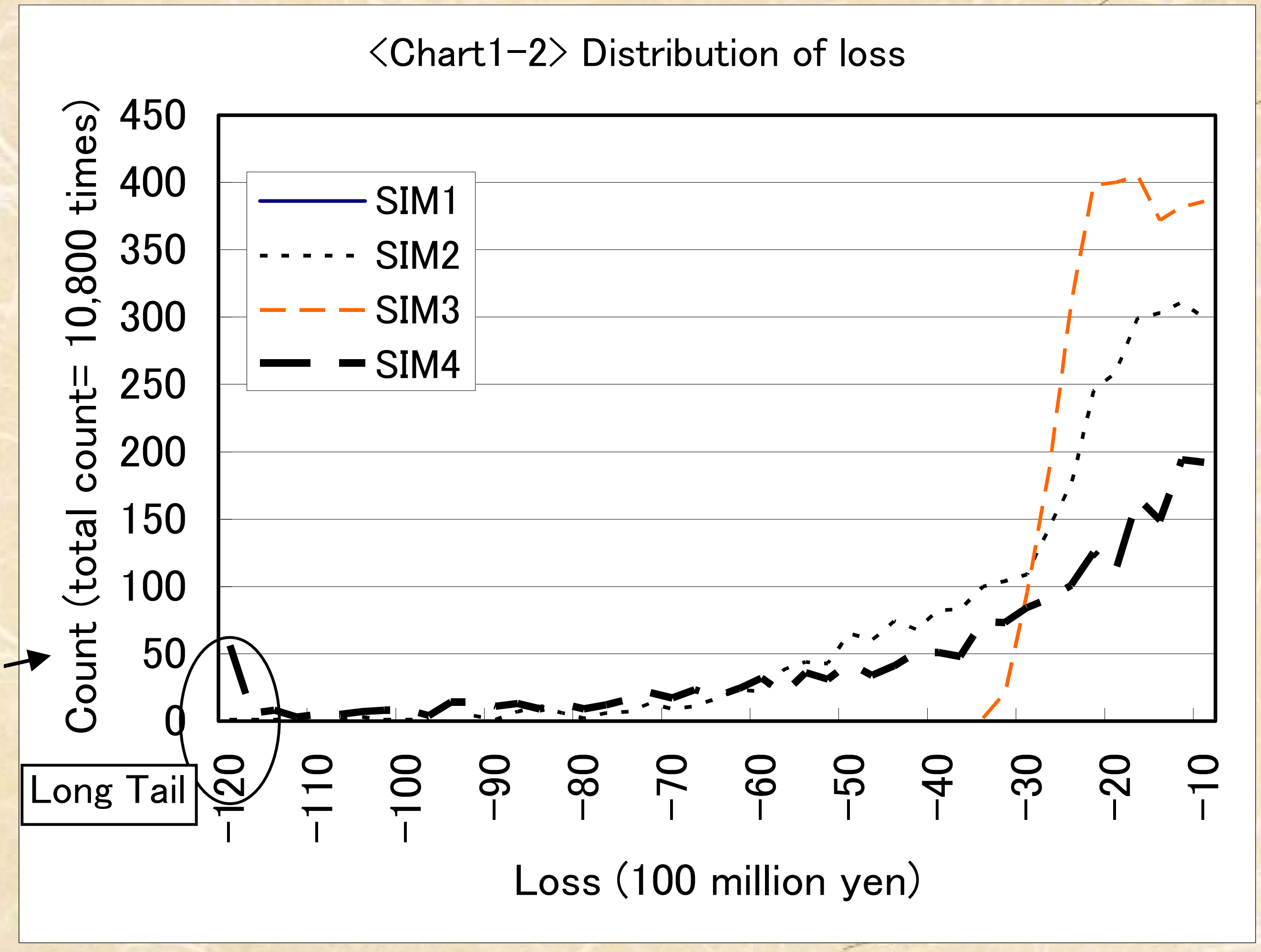
	Symbol	(SIM1) Standard Model			(SIM2) Model A		
		Formula(1)	Sim(2)	(2)/(1)	Formula(1)	Sim(2)	(2)/(1)
N=21,000、Insured amount ＝1							
Expected value of loss	E[L]	0.00000	0.00000	–	0.00000	0.00000	–
Variance of loss	Var[L]	719.66392	711.03772	98.80%	45,290.30538	44,873.24145	99.08%
Standard deviation of loss	Std[L]	26.82655	26.66529	99.40%	212.81519	211.83305	99.54%
Variance of loss per policy	Var[L/N]	0.03427	0.03386	98.80%	2.15668	2.13682	99.08%
Standard deviation of loss per policy	Std[L/N]	0.00128	0.00127	99.40%	0.01013	0.01009	99.54%
N=21,000、Insured amount ＝10 mil yen							
Expected value of loss	E[L]	0	0	–	0	0	–
Variance of loss	Var[L]	7.19664E+16	7.11038E+16	98.80%	4.52903E+18	4.48732E+18	99.08%
Standard deviation of loss	Std[L]	268,265,525	266,652,905	99.40%	2,128,151,907	2,118,330,509	99.54%
Variance of loss per policy	Var[L/N]	3.42697E+12	3.38589E+12	98.80%	2.15668E+14	2.13682E+14	99.08%
Standard deviation of loss per policy	Std[L/N]	12,775	12,698	99.40%	101,341	100,873	99.54%

	Symbol	(SIM2) Model B1			(SIM2) Model B2		
		Formula(1)	Sim(2)	(2)/(1)	Formula(1)	Sim(2)	(2)/(1)
N=21,000、Insured amount ＝1							
Expected value of loss	E[L]	0.00000	0.00000	–	0.00000	0.00000	–
Variance of loss	Var[L]	19,950.06964	19,929.35301	99.90%	55,641.62648	55,635.59084	99.99%
Standard deviation of loss	Std[L]	141.24472	141.17136	99.95%	235.88477	235.87198	99.99%
Variance of loss per policy	Var[L/N]	0.95000	0.94902	99.90%	2.64960	2.64931	99.99%
Standard deviation of loss per policy	Std[L/N]	0.00673	0.00672	99.95%	0.01123	0.01123	99.99%
N=21,000、Insured amount ＝10 mil yen							
Expected value of loss	E[L]	0	0	–	0	0	–
Variance of loss	Var[L]	1.99501E+18	1.99294E+18	99.90%	5.56416E+18	5.56356E+18	99.99%
Standard deviation of loss	Std[L]	1,412,447,154	1,411,713,605	99.95%	2,358,847,737	2,358,719,798	99.99%
Variance of loss per policy	Var[L/N]	9.50003E+13	9.49017E+13	99.90%	2.6496E+14	2.64931E+14	99.99%
Standard deviation of loss per policy	Std[L/N]	67,259	67,224	99.95%	112,326	112,320	99.99%

[8. Simulation Example]



[8. Simulation Example]



[8. Simulation Example]

- Standard deviation of loss per policy per annual net premium

Symbol	(SIM1) Std Model	(SIM2) Model A	(SIM3) Model B1	(SIM4) Model B2
$\frac{1}{\overline{P}_x^*} Std[\frac{{}^{Port}L^*}{n}]$	0.101	0.782	0.522	0.871
$2.5\frac{1}{\overline{P}_x^*} Std[\frac{{}^{Port}L^*}{n}]$	0.252	1.954	1.304	2.178

- To cover 99.3% of the risk, we need 2.5 times of standard deviation
  - For standard model, 3 months of premium (or smaller if larger portfolio)
  - 2 years of premium for Model A or B2 seems to be significantly large

[9. Conclusion]

- Influenza pandemic occurs periodically in every several decades.
- Once it happens, its impact to our human lives might be huge and severe.
- Loss stochastic variable (L) of an insurance portfolio is considered and its standard deviation is obtain in a theoretical way.
- One advantage of our approach, compared to stochastic simulation is its convenience. Our approach can compliment the stochastic simulation approach.

Thank you very much for your attention!

END.