Influenza Pandemic

and an Actuarial Model That Takes Account Additional Mortality

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Abstract

Influenza pandemic would be defined as a situation when a new influenza virus emerges and infects humans, starts to spread easily and sustainably among humans, and finally causes widespread epidemic among the world. This is a risk (pandemic risk) that might cause worldwide increase of death and social/economic confusion. This risk has been strongly recognized among medical authorities; however, in the last few years, general public also have heightened their sense of crisis. This article is an effort to address this risk from actuarial perspectives.

In this article, the author first summarizes the general information regarding current understanding of influenza pandemic in the world. Then the author explains the risk characteristics from actuarial point of view by comparing it with some other types of risk (for example, earthquake risk as a catastrophic risk). In later chapter, the author introduces two types of life contingency models which extend traditional actuarial mathematics and which can be used to analyze pandemic and other catastrophe type mortality risks. For whole life policies, the author shows some closed formula of some actuarial notations, such as single premium and net level premium rate. After that, the author considers a life insurance portfolio that consists of n policies. Its present value of loss is examined as a measure of mortality risk of the portfolio. This article shows an approach of calculating a theoretical value of the standard deviation of the loss and it is compared with a stochastic simulation result.

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(Chapter 1) Introduction

(1-1) Influenza pandemic would be defined as a situation when a new influenza virus emerges and infects humans, starts to spread easily and sustainably among humans, and finally causes widespread epidemic among the world. This is a risk (pandemic risk) that might cause worldwide increase of death and social/economic confusion. This risk has been strongly recognized among medical authorities; however, in the last few years, general public also have heightened their sense of crisis. This article is an effort to address this risk from actuarial perspectives.

(1-2) In Chapter 2, the author first summarizes the general information regarding current understanding of influenza pandemic in the world. There are already many well developed reports from governmental organizations and articles written for general public available. By reading these materials, we understand that we face a critical situation regarding the influenza pandemic. In Chapter 3, the author explains risk characteristics from actuarial point of view by comparing it with some other types of risk (for example, earthquake risk as a catastrophic risk). Our main concern in this article is mortality risk, while other risks related to the influenza pandemic is not addressed in this paper.

(1-3) Chapter 4 starts with a brief review of traditional, standard life contingencies, and then we discuss two types of models that incorporate excess mortality caused by pandemic or other catastrophic type mortality risks. These models are extensions of the traditional life contingencies and would be used to analyze these risks. Using whole life policies, some closed formula of some actuarial notations, such as single premium and net level premium rate are obtained. Later in this chapter, we consider an insurance portfolio that consists of n policies. It is well known that, if we assume such excess mortality, the mortality risk of the portfolio is not diversifiable. We examine the present value of loss stochastic variable of the portfolio closely. Taking its standard deviation as a risk measure of the portfolio, this article shows an approach of calculating the theoretical value of the standard deviation in spreadsheets. In Chapter 5, the author developed a stochastic simulation model of the insurance portfolio, and compares the simulated value with the theoretical value determined based on Chapter 4 result. By performing stochastic simulation, we can obtain the distribution of the loss, while the advantage of our theoretical approach is its convenience of calculating the standard deviation. Such easy to use approach can be useful in risk management situation, for example, for a solvency management issues. Finally, Chapter 6 provides some conclusions and remarks.

(1-4) The opinion presented here is solely based on the author's thoughts and would not represent the opinion by the company or the actuarial organization the author belongs to. Also all the errors and misstatement is the responsibility of the author and not of the references.

(Chapter 2) Influenza Pandemic

(2-1) In this chapter, we will look through a basic knowledge of influenza pandemic which might be useful as a background for later chapter discussion. Since there are many well developed reports and articles available, please refer to these sources if you have interested in this subject.

(2-2) What is influenza pandemic?
In this paper, we define influenza pandemic as a situation when a new influenza virus emerges and infects humans, starts to spread easily and sustainably among humans, and finally causes widespread epidemic among the world. This is a risk (pandemic risk) that might cause worldwide increase of death and social/economic confusion.

(2-3) What makes the influenza pandemic as a big threat to our society?
It is predicted that such an emergence and spread of new types of influenza will occur periodically in every several decades. Once it happens, such epidemic might become a world wide phenomenon. Past experience in 1918 of the catastrophic spread of Spanish influenza caused disastrous loss of human lives and we might have a similar or even larger loss in the next pandemic. Such worst case scenario is not a grandiose figment: some type of new influenza virus, called H5N1 type virus, is highly pathogenic. Current development of transportation network has enabled the influenza spread much faster and among the continents. These changes of circumstance tell that the past experience may not be applied in the current situation. We must take in mind that the current situation does not allow optimism.

(2-4)Influenza occurred in the 20^{th} century
There are three influenza pandemics occurred and recorded in the 20^{th} century:

1) Spanish Influenza (1918-1919): Total number of the patient counted 25-30% of the total population of the world. Case fatality rate (death over the infected) is more than 2.5%. In Japan, the number of patient is estimated as 23 million and the 380,000 people died ([15]). Virus type is H1N1.
2) Asian Influenza (1957-1958): Case fatality rate seems to be much lower than that of Spanish Influenza. Estimated excess number of death is more than 2 million people worldwide. Virus type is H2N2.
3) Hong Kong Influenza (1968-1969): The impact is even milder than that of Asian Influenza. Virus type is H3N2.

(2-5)How would the influenza spread among the people and cause pandemic?
There are several stages before new influenza causes pandemic situation. WHO classifies these stages into 6 phases. Following is a simplified explanation of such stages.

- A situation is that there exist new (potential) types of avian influenza in birds (wild birds or poultry); however, there is no infection to human (WHO stage 1 and 2).
- A situation is that infection from birds to human is sometimes reported; however, the infection from human to human is not recognized (WHO stage 3, which is a current situation).
- A situation is that infection from human to human is recognized; however, the infection is still limited and under control (WHO stage 4 and 5)
- A situation is that infection from human to human is done in an efficient and sustained manner and that influenza starts to spread(WHO stage 6, or pandemic)

When an influenza virus obtains a new infection power, genetic mutation occurs in a virus. Such mutations can be either as an antigenetic drift or an antigenetic shift. The former refers to a continuous, evolutional mutation process in a virus, while the latter refers to a sudden, evolutional mutation process. What is known as genetic assortment is a type of antigenetic shift.

(2-6) What is influenza virus? What is highly pathogenic and weakly pathogenic virus?

Influenza virus can be classified into three types, type A, B and C, by the difference of the protein in the virus. Among then, type influenza is known to cause influenza pandemic. Following explanation refers to type A influenza.

On a surface of influenza virus, there exist two types of proteins that outshoot from the surface. One is called hemagglutinin (HA) and the other is called neuraminidase (NA). There are 16 subtypes of HA (H1 to H16) and 9 subtypes of NA (N1 to N9). They have critical rolls in infecting and duplication process. The pathogenicity of influenza depends on the HA protein. H5 and H7 subtype is known as highly pathogenic, since once virus with these subtypes infect human, it can attack human organ directly in the body and therefore, infected person tends to show severe symptom and the fatality rate is high. Virus with other HA subtypes are known as low pathogenic. All 20th century pandemics were caused by low pathogenic virus, while the new influenza viruses found in Asian countries recently are H5N1 subtype, which foster the sense of crisis.

(2-7) Possible impact of pandemic

Possible impact of influenza pandemic should be understood from various perspectives. It has impact on human lives, our economy and the society. Also, there are both long-term as well as short-term impacts.

The report by the Infectious Disease Surveillance Center of Japan ([15]) provides the following list of impact. 1) A huge number of patient and death, 2)mental and physical pain, 3) infection to the medical practicians, 4) excess demand to medical institutions and breakdown of medical services, 5) Infection to the work staffs of social infrastructure (transportation, communication, police, electric, food, water ,fire, etc.), 6) a breakdown of social function and public administration, 7) restriction on daily activities, 8) restriction on corporate activities, 9) collapse of the foundation of corporation, due to the domino effect, 10) decrease of population in production age, and 11) a huge economical loss.

The level of impact is affected by various factors. According to the above report, such factor include, 1) pathogenicity of the virus itself, 2) number of infected in epidemic period, 3) proportion of severe complication, 4) Distribution of patients' age, 5) speed of the pandemic spread, 6)the degree of preparation, 7) effect of the action, 8) social background and infrastructure (age distribution of the population, population density, sanitary condition in general, and 9) level of the medical treatment.

With respect to the estimate of human and economic impact, following are some citations from reports prepared by governmental organizations.

<Impact on Human lives>

Ministry of Health, Labour and Welfare of Japan has estimated a number of deaths of the Japanese ([7]). The model they used was originally developed by the Center of Disease Control (CDC) of the U.S.A ([10]). Using that model, it is estimated that the 25% of the total population of Japanese will be infected by the influenza virus. The fatality rate is assumed to be 0.53% for moderate cases (similar to Asian Influenza) and 2% for severe cases (similar to Spanish Influenza). The number of deaths is estimated 170 thousand for severe cases and 640 thousands for severe cases. The report remarks that effects of the following factors such as intervention with new influenza vaccines or Antiviral drugs and sanitation conditions in Japan are not taken consideration in these estimates.

<Impact on the Economy >

The reports prepared by the Congressional Budget Office of the U.S.A ([8],[9]) has concluded that the overall impact of a potential flu pandemic on gross domestic product to be about 4.25% in a severe scenario and about 1% in the mild scenario. The reports explain that severe scenario refers to the one similar to Spanish influenza and the mild scenario refers to the one similar to Asian or Hong Kong influenza.

(Chapter 3) Pandemic Risk

(3-1) In this chapter, we discuss the risks that insurers bear when an influenza pandemic occurs. We focus especially on mortality risks.

(3-2) From insurers' perspective, pandemic risk is defined as the risk that occurs when influenza pandemic emerges and the excess claim may cause insurers' insolvency.

(3-3) Pandemic risk may classified as a kind of catastrophe risk (or calamity risk) like earthquake risk. There are several differences between earthquake and pandemic risk. Following chart summarizes the difference between pandemic and some other risk.

<Chart 1: Comparison of mortality risk>

Item	Pandemic	Earthquake	War/Terrorism	Life underwriting risk in general
Area	Might be global phenomenon	Local	Assumed Local (*1)	—
Time horizon	Several weeks to several years	In a moment to several days	Several days to several decades	—
Cycle Length	Once in several decades	Once in several decades per one area	Unpredictable	Death occurs continuously
Impact on Human/Property	Human	Both Human/Property	Both Human/Property	Human
Impact by age	Might take various patterns	Uniform among ages	Might take various patterns	As shown in mortality table
Distribution of loss	Distribution with a long tail	Distribution with a long tail	Unknown	Normal distribution (Law of large numbers)
Quantitative evaluation of loss	Difficult (*2)	Some experience in PC insurers	Difficult	Sufficient experience in life insurers
Risk diversifiability	Not diversifiable	Not diversifiable	Not diversifiable	Diversifiable
Approach to risk	Contingency reserve, Surplus, Reinsurance, Securitization	Contingency (or catastrophe) reserve, Surplus, Reinsurance, Securitization	Exclusion clause	Law of large numbers, Risk selection, Contingency reserve, Surplus, Reinsurance, Securitization

(*1) Global war is not likely to occur

(*2) Research has just started

<Chart 2> Comparison between pandemic risk and earthquake risk

	Similar issues	Different issues
Pandemic risk	- Catastrophe (Calamity) risk -Occurs periodically in every several decades - Have a distribution of long-tail - Risk is not diversifiable - Exclusion clause might be applied -Need government level preparation to prevent/ease losses -(Currently in Japan) Contingency reserve I might be used	- Might be global (worldwide) in a long term (might be local in a short-term) - The affect may continue for more than several years - Different impact on humans by their ages - Mainly impact on human lives - Early stage counter-action might be critical to prevent /ease the pandemic - Quantitative risk evaluation might be difficult (Model risk and parameter risk might be significant)
Earthquake risk		- Limited to local or in a area - Occurs in a very short period (several days) - Impact on all humans in a same degree - Impact on both human lives and property - Some experience of quantitative risk evaluation

(3-4) As summarized in Chart 2, there are several issues that are not similar between pandemic risk and earthquake risk. First, earthquake risk is limited in relatively local region and the damage is limited in an area of, say, several hundred kilo-meters radius, while for pandemic risk is truly a global phenomenon. In other words, the earthquake risk is geometrically diversifiable while the pandemic risk is not. Once the influenza pandemic spreads all around the world, the impact might be huge.

There are several patterns of how the pandemic affects human by age. If we classify the human into three categories by age, childhood (age 0 to 18), adulthood (19-64), and elderly (65 and above), the impact of pandemic might be different. Following are some typical patterns:

Pattern 1) Childhood and elderly people tends to be affected relatively severely by the influenza while adulthood tends to be affected relatively mildly. This pattern implies that the former classes are thought to be less resident to influenza to the latter class.
Pattern 2) Adulthood people tends to be affected relatively severely by the influenza while childhood and elderly tends to be affected relatively mildly. One explanation is that once an adult person, who has more resident to childhood and elderly, comes down with flu, the resident system of the body overreacts and produce internal substance called cytokine more. Such over-reaction has the opposite effects of making the illness more severe. (This phenomenon is called cytokine storm). Also, since adulthood people contacts with other person much frequently in a society, than childhood and elderly people, they has more opportunity to be affected by the influenza.
Pattern 3) Impact of influenza is same among childhood, adulthood and elderly people. This is the same situation as the impact of earthquake risk.

It is difficult to predict which pattern will emerge. Also, the pattern may be affected by the governmental/social reaction to the pandemic at the beginning of the influenza spread. Temporary closing of school classes and business offices, for example may be effective to prevent the widespread of influenza.

Risk quantification of pandemic risk is difficult and the author believes it is in an early stage, compared to the risk quantification of earthquake risk. For pandemic risk, the actual data is very limited. When we develop a risk model, model risk and parameter risk might be significantly large and always have to be taken care.

(3-5) Approaches to the mortality risk management

For standard mortality risk, with which we expect that the death occurs randomly and independently, insurance company can diversify the mortality risk by constructing the insurance portfolio as uniform as possible. For instance, insurers may limit the minimum and maximum amount of death benefit, or may apply strict risk selection procedures. This is a traditional approach which utilizes the law of large numbers.

In addition, for a sudden, unexpected risk other than usual death, contingency reserves and surplus funding are in practice to ensure the solvency on both mandatory and voluntary base. Taking the contingency reserve regulation in Japan, life insurers must fund contingency reserve I to cover the mortality risk for ordinary life coverage. Regulatory maximum contingency reserve I requirement for ordinary death is 0.6 permillage of insurance amount at risk. This coefficient might be re-estimated once we have established a well developed method to quantify pandemic risk.

Reinsurance, especially non-proportional type reinsurance such as stop-loss reinsurance can be used to prepare unexpected risk. Regarding the pandemic risk, reinsurance may not work; however, if the pandemic become a global phenomena and the geometrical diversification may not work. As an advanced version of reinsurance, securitization of mortality risk is currently gaining attention.

(3-6) Approaches to risk quantification

To prepare for the pandemic risk, we need any kind of risk quantification. There are several practical, but strong approaches. One approach is a scenario setting, another approach is stochastic simulation. In this paper, the author takes theoretical approach.

Approaches to be taken depend on the risk measure taken for risk quantification. In this paper, we take loss stochastic variable of an insurance portfolio and examine its characteristics. Variance, value at risk (VaR) or tail value at risk (TVaR) are some well known measures for risk quantification. Simulation is the only approach to calculate VaR or TVaR; however, developing sophisticated risk model is time consuming and need lot of resources. This paper's approach is focus mainly on variance, and therefore, there is a limitation; however, it is simple and does not need model development. In practice, several approaches are mixed and used together.

(Chapter 4) Extended Actuarial Model That Reflects Additional Mortality Caused by Pandemic

(4-1) In this section, we examine two actuarial models that include additional mortality caused by a pandemic or some catastrophic events. We start by reviewing some basic results of traditional actuarial mathematics. We see some actuarial expressions for continuous pay whole life. After that, we will extend these results by including additional mortality caused by sudden death by pandemic or other catastrophic events.

<A review of traditional life contingencies>

(4-2) We define some notations as follows;

$$T = T(x) \text{ : } stocastic \text{ var} iable \; that \; means \; future \; lifetime \; of \;\; a \; life \; of \; age(x)$$

$$F_T(t) = \Pr[T \leq t] : Cummulative \; probability \; distribution \; funcition \;\; of \; T$$

$$f_T(t) = \frac{\partial F_T(t)}{\partial t} : \Pr obability \; density \; function \; of \; T$$

$$\mu(x+t) = -\frac{1}{F_T(t)}\frac{\partial F_T(t)}{\partial t} = \frac{f_T(t)}{F_T(t)} : Force \; of \; mortality$$

We also define survival function as $_t p_x = 1 - F_T(t)$. Using this, we have following expression,

$$_t p_x = 1 - F_T(t) = \exp(-\int_0^t \mu(x+s)ds)$$

$$f_T(t) = {_t p_x} \mu(x+t)$$

We assume that a set of continuous mortality is provided a priori by one of the following way; a force of mortality, a cumulative distribution function of T, a probability density function of T, survival function, or other equivalent set of information.

(4-3) We discuss in the continuous time framework; however, if we are given a mortality table $\{q_x\}$ (x=0,1,2,・・・), which is a discrete mortality information, we will convert mortality rate of each age into force of mortality by assuming a uniform distribution of death between age x+t and x+t+1. In other words, for x and t as integer and for 0≦s＜1, we assume that,

$$\mu(x+t+s) = \frac{\mathrm{q}_{\mathrm{x+t}}}{1 - \mathrm{sq}_{\mathrm{x+t}}},$$

And we have,

$$_s p_{x+t} = 1 - sq_{x+t}$$

(4-4) Take continuous premium pay, immediate benefit pay whole life policy into consideration. Traditional life mathematics is developed by defining some stochastic variables such as Z (present value of future benefit payment) or L (present value of future loss by the company) from T.

Let,

$$Z = e^{-\delta T} \text{ : } \quad Stochastic \text{ var} iable \; that \; represents \; present \; value \; of \;\; future \; benefit \; payment$$

Then, we can derive some actuarial functions using Z.

・ Single premium, present value of life annuities, and variance of Z

$$\overline{A}_x = E[Z] = \int_0^{\omega-x} e^{-\delta t}\,{}_t p_x \mu(x+t)dt = \frac{i}{\delta}\sum_{t=0}^{w-x} e^{-\delta(t+1)}\,{}_{t|}q_x = \frac{i}{\delta}A_x$$

$$Var[Z] = {}^{2\delta}\overline{A}_x - (\overline{A}_x)^2$$

$$\overline{a}_x = E[\overline{a}_{\overline{T|}}] = \int_0^{\omega-x} e^{-\delta t}\,{}_t p_x dt = \frac{1-\overline{A}_x}{\delta}$$

$$Var[\overline{a}_{\overline{T|}}] = \frac{1}{\delta^2}({}^{2\delta}\overline{A}_x - (\overline{A}_x)^2)$$

Symbols shown on the upper left of a notation represents a force on interest used for discounting, for example,

$${}^{2\delta}\overline{A}_x = E[\exp(-2\delta T)] = \int_0^{\omega-x} e^{-2\delta t}\,{}_t p_x \mu(x+t)dt = \frac{e^{2\delta}-1}{2\delta}\sum_{t=0}^{w-x} e^{-2\delta(t+1)}\,{}_{t|}q_x = \frac{e^{2\delta}-1}{2\delta}\,{}^{2\delta}A_x .$$

2 δ means to use 2 δ instead of δ to discount future benefit payment. In other words, use discounting factor of $e^{-2\delta t}$ instead of $e^{-\delta t}$ in the integral above.

・ Net premium rate

$$\overline{P}_x = \frac{\overline{A}_x}{\overline{a}_x} = \frac{\delta\overline{A}_x}{1-\overline{A}_x}$$

represents the net premium rate for this whole life policy.

・ Future loss stochastic variable (L) and its variance

Using $\overline{P}_x$, a stochastic variable L is defined by,

$$L = e^{-\delta T} - \overline{P}_x \times \overline{a}_{\overline{T|}} .$$

L represents present value of future loss of insurance company at policy issue. L is the difference between present value of future benefit payment and present value of future premium income. Since net premium is defined using the equivalence principle of profit and loss, E[L]=0. Also, we have the following expression,

$$L = \frac{(\delta+\overline{P}_x)Z - \overline{P}_x}{\delta}$$

and the variance of L is shown as,

$$Var[L] = (1+\frac{\overline{P}_x}{\delta})^2 Var[Z] = (1+\frac{\overline{P}_x}{\delta})^2({}^{2\delta}\overline{A}_x - (\overline{A}_x)^2) .$$

(4-5) Next, we consider an insurance portfolio that consists of n policies, issued at the same time (t=0) for n insured, all age (x). Insurance amount is 1. All insured follow the same mortality assumption and death occurs randomly and independently. Termination other than death is not assumed.

It is well known that the mortality risk for this insurance portfolio is diversifiable. Mathematically, this fact is formalized in the following way,

Let present value of benefit payment and present value of future loss for the kth (k=0,1,2…n) policy be Z_k and L_k, respectively. Let the total of L_k be ${}^{Port}L$. ${}^{Port}L$ represents the future loss stochastic variable for the insurance portfolio. We assume that the death occurs independently. In other words, Z_k and Z_l ($k \neq l$) are independent and the covariance of Z_k and Z_l is zero.

Then,

$$L_k = \frac{(\delta + \overline{P}_x)Z_k - \overline{P}_x}{\delta}$$

$$Var[L_k] = (1 + \frac{\overline{P}_x}{\delta})^2 Var[Z]$$

$$Cov[L_k, L_l] = (1 + \frac{\overline{P}_x}{\delta})^2 Cov[Z_k, Z_l] = 0 \ (k \neq l)$$

$${}^{Port}L = L_1 + L_2 + \Lambda\ L_n$$

$$Var[{}^{Port}L] = nVar[L] + \sum_{k \neq l} Cov[L_k, L_l]$$

$$= n(1 + \frac{\overline{P}_x}{\delta})^2 Var[Z]$$

The future loss of the insurance portfolio per policy (${}^{Port}L/n$) converges to zero, as n goes to infinity. In other words, the mortality risk is diversifiable.

$$Var[\frac{{}^{Port}L}{n}] = \frac{1}{n}(1 + \frac{\overline{P}_x}{\delta})^2 Var[Z] \longrightarrow 0 \qquad (n \longrightarrow \infty)$$

<Description of the models>

(4-6) Now we consider catastrophe type risks such as pandemic risk or earthquake risk and try to include these risks in traditional life contingencies. In the following two models, it is assumed that additional death caused by a catastrophe is assumed, in addition to usual death assumption which is reflected in mortality table or force of mortality. More specifically, it is assumed that when a catastrophe event occurs, 1-R of the total population would die immediately while the rest (R) survives after the event. R might be either a constant value or a stochastic value, depending on the model. Some detailed assumptions are added to have the following two types of the models (Model A and Model B). In developing these models, the author refers to Hu and Cox [4] for their ideas.

(4-7) Model A

In model A, we assume that a catastrophic event occurs once in $1/\lambda$years in average. When the event occurs, 1- r of the population dies and the rest (r) survives. Here λ and r are assumed to be a constant. Catastrophic events occur not only once. The event occurs with the same probability in the future. In other words, the interval stochastic variable, U_1, U_2... between two the events follows an exponential distribution with average $1/\lambda$. Mathematically written, cumulative probability distribution functions are shown as,

$$F_U(u) = 1 - \exp(-\lambda u)$$

where U_1, U_2, ... are independent each other.

It is well known that, if there are K catastrophic event occurred in time 0 to t, K follows Poisson distribution with average λt;

$$\Pr[K = k] = e^{-\lambda t} \frac{(\lambda t)^k}{k!}.$$

(4-8) Model B

In model B, catastrophic event occurs only once. It assumes to occur in 1/λyears in average. When the event occurs, 1-R of the population dies and the rest (R) survives. Here λ is a constant but R is assumed to be a stochastic variable. R takes a value between 0 and 1, and E[R]=r, Var[R]=σ^2. Stochastic variable U that represents the time of stochastic event follows exponential distribution with average 1/λ.

<Closed expressions of some actuarial functions>

(4-9) For model A, some actuarial expressions such as single premium are obtained in a closed form. In the following notations, star signs (^{*A,*B}) on the upper right implies model A or model B (^{A,B} may be omitted).

Let $T^{*A}(x)$ be future lifetime stochastic variable in model A. We will obtain cumulative probability distribution function, $F_{T^*}(t)$. Assume that catastrophic event occurs K (=0,1,2…) times until time t,

$$F_{T^*}(t) = \Pr[T^* \le t] = 1 - \sum_{k=0}^{\infty} \Pr[T^* > t | K = k] \times \Pr[K = k].$$

Here $\Pr[T^* > t | K = k]$ implies the probability that the insured survives at time t, assuming that the event occurred k times,

$$\Pr[T^* > t] = r^k \times {}_t p_x.$$

Since K follows Poisson distribution with average λt,

$$\begin{aligned} F_{T^*}(t) &= 1 - \sum_{k=0}^{\infty} r^k \times {}_t p_x \times e^{-\lambda t} \frac{(\lambda t)^k}{k!} \\ &= 1 - {}_t p_x \times e^{-\lambda t} \sum_{k=0}^{\infty} \frac{(r\lambda t)^k}{k!} \\ &= 1 - {}_t p_x \times e^{-\lambda(1-r)t} \end{aligned}$$

Therefore,

$$\mu^*(x+t) = \mu(x+t) + \lambda(1-r)$$

$$f_{T^*}(t) = {}_t p_x \times \mu(x+t) \times e^{-\lambda(1-r)t} + \lambda(1-r) \times {}_t p_x \times e^{-\lambda(1-r)t}$$

It is worth to comment on the expression of force of mortality shown above. This expression implies that the force of mortality is given by an assumed mortality $\mu(x+t)$ plus additional constant mortality that represents the probability of catastrophic event. Compare with a situation of additional constant mortality for an insured that has hazard occupation such as a pilot. He or she faces additional mortality, which threatens his or her life continuously. The expression of force of mortality doesn't distinguish the nature of the risk. In other words, we can't judge the nature of the risk by just looking at the expression of the force of mortality. We will see later that the additional mortality for hazard occupation is diversifiable while that for catastrophe is not diversifiable.

(4-10) Next, define Z^* as, $Z^*=\exp(-\delta T^*)$, and

$$\begin{aligned}
\overline{A}_x^* &= E[Z^*] \\
&= \int_0^{\omega-x} e^{-\delta t} \times f_{T^*}(t)dt \\
&= \int_0^{\varpi-x} e^{-\delta t} \times ({}_tp_x \times \mu(x+t) \times e^{-\lambda(1-r)t} + \lambda(1-r) \times {}_tp_x \times e^{-\lambda(1-r)t})dt \\
&= \int_0^{w-x} e^{-(\delta+\lambda(1-r))t} \times {}_tp_x \times \mu(x+t)dt + \lambda(1-r)\int_0^{w-x} e^{-(\delta+\lambda(1-r))t} \times {}_tp_x dt
\end{aligned}$$

Here we introduce the following notation,

$$ {}^{\delta+\lambda(1-r)}\overline{A}_x = \int_0^{w-x} e^{-(\delta+\lambda(1-r))t} \times {}_tp_x \times \mu(x+t)dt $$

$$ {}^{\delta+\lambda(1-r)}\overline{a}_x = \int_0^{w-x} e^{-(\delta+\lambda(1-r))t} \times {}_tp_x dt $$

This notation represents the single premium for a whole life for T(x), replacing the force of interest from δ to $\delta+\lambda(1\text{-}r)$. Note that ${}^{\delta}\overline{A}_x = \overline{A}_x$, ${}^{\delta}\overline{a}_x = \overline{a}_x$.

Since we have the following relation,

$$1={}^{(\delta+\lambda(1-r))}\overline{A}_x + (\delta+\lambda(1-r))\,{}^{(\delta+\lambda(1-r))}\overline{a}_x$$

we have,

$$\begin{aligned}
\overline{A}_x^* &= {}^{\delta+\lambda(1-r)}\overline{A}_x + \lambda(1-r) \times {}^{\delta+\lambda(1-r)}\overline{a}_x \\
&= 1 - \delta \times {}^{\delta+\lambda(1-r)}\overline{a}_x \\
&= \frac{1}{\delta+\lambda(1-r)}\left(\lambda(1-r) + \delta \times {}^{\delta+\lambda(1-r)}\overline{A}_x\right)
\end{aligned}$$

Also, $1 = \overline{A}_x^* + \delta \times \overline{a}_x^*$ and

$$\overline{a}_x^* = {}^{\delta+\lambda(1-r)}\overline{a}_x \, .$$

We have the expression of the variance of the present value of benefit payment as,

$$
\begin{aligned}
Var[Z^*] &= E[(Z^*)^2] - (E[Z^*])^2 \\
&= {}^{2\delta+\lambda(1-r)}\overline{A}_x + \lambda(1-r)\times {}^{2\delta+\lambda(1-r)}\overline{a}_x - (\overline{A}^*_x)^2 \\
&= \cdots \\
&= (1 - 2\delta\times {}^{2\delta+\lambda(1-r)}\overline{a}_x) - \left(1 - \delta\times {}^{\delta+\lambda(1-r)}\overline{a}_x\right)^2 \\
&= \frac{1}{2\delta+\lambda(1-r)}\left(\lambda(1-r) + 2\delta\times {}^{2\delta+\lambda(1-r)}\overline{A}_x\right) - \left(\frac{1}{\delta+\lambda(1-r)}\left(\lambda(1-r) + \delta\times {}^{\delta+\lambda(1-r)}\overline{A}_x\right)\right)^2
\end{aligned}
$$

Also, if we define the present value of future loss stochastic variable as, $L^* = \exp(-\delta T^*) - \overline{P}^*_x \times \overline{a}_{\overline{T^*|}}$,

where $\overline{P}^*_x$ be a net premium rate,

$$
\overline{P}^*_x = \frac{\overline{A}^*_x}{\overline{a}^*_x} = \frac{\delta \overline{A}^*_x}{1-\overline{A}^*_x}
$$

we have the variance of loss stochastic variable as,

$$
Var[L^*] = (1 + \frac{\overline{P}^*_x}{\delta})^2 \times Var[Z^*].
$$

(4-10) Also for model B, we can express some actuarial expressions such as single premium in a closed form. First, we consider the situation where R=r (constant). In a similar way as model A,

$$
F_{T^*}(t) = \Pr[T^* \le t] = \sum_{k=0}^{1} \Pr[T^* \le t \mid K = k] \times \Pr[K = k].
$$

Here K=0 implies that the event has not been occurred yet, while K=1 implies that the event has already occurred.

$$
\begin{aligned}
F_{T^*}(t) &= \Pr[T^* \le t \mid K = 0] \times \Pr[K = 0] + \Pr[T^* \le t \mid K = 1] \times \Pr[K = 1] \\
&= (1 - {}_tp_x) \times e^{-\lambda t} + (1 - r \times {}_tp_x) \times (1 - e^{-\lambda t}) \\
&= 1 - {}_tp_x \times (r + (1-r)e^{-\lambda t})
\end{aligned}
$$

For a random variable R in general, let the probability density function of R as, $g_R(s)$ and let E[R] =r,

$$
\begin{aligned}
F_{T^*}(t) &= \int_0^1 g_R(s) \times (1 - {}_tp_x \times (s + (1-s)e^{-\lambda t})ds \\
&= 1 - {}_tp_x \times (r + (1-r)e^{-\lambda t})
\end{aligned}.
$$

The cumulative probability distribution function has the same form. In other words, above expression does not rely on the distribution of R, but only rely on the expected value of R (=r). By taking the derivative in the right side, we have,

$$
f_{T^*}(t) = {}_tp_x \times \mu_{x+t} \times (r + (1-r)e^{-\lambda t}) + {}_tp_x \times (1-r)\lambda e^{-\lambda t}
$$

Then we have,

$$\overline{A}_x^* = r\overline{A}_x + (1-r)^{(\delta+\lambda)}\overline{A}_x + \lambda(1-r)\times^{(\delta+\lambda)}\overline{a}_x$$
$$= r\overline{A}_x + (1-r)\times(\frac{\lambda}{\delta+\lambda} + \frac{\delta}{\delta+\lambda}{}^{(\delta+\lambda)}\overline{A}_x)$$

Using the relation, $1 = \overline{A}_x^* + \delta\times\overline{a}_x^*$,

$$\overline{a}_x^* = r\times\overline{a}_x + (1-r)\times^{(\delta+\lambda)}\overline{a}_x$$

For reference, above single premium formula can be obtained in the following way. First, assume that a catastrophic event occurs at time τ, which is a constant value. "r" is assumed to be constant for a time being.

$$\overline{A}_x^* = \overline{A}^{\,1}_{x:\overline{\tau}|} + e^{-\delta\tau}\times{}_\tau p_x((1-r) + r\overline{A}_{x+\tau})$$
$$= (1-r)\overline{A}_{x:\overline{\tau}|} + r\overline{A}_x$$

Above expression is a weighted average of a single premium of endowment life and a single premium of a whole life. When τ is not a constant but has the exponential distribution with average 1/λ, the first term turns out to be,

$$\int_0^\infty \overline{A}_{x:\overline{\tau}|}\times\lambda e^{-\lambda\tau}d\tau = \Lambda$$
$$= \frac{\lambda}{\delta+\lambda} + \frac{\delta}{\delta+\lambda}{}^{(\delta+\lambda)}\overline{A}_x$$

and we have the expression in (5-10). Same argument is valid when r is a random variable R.

(4-11) We have closed expressions for Var[Z*] and Var[L*].

$$Var[Z^*] = E[(Z^*)^2] - (E[Z^*])^2$$
$$= r\times^{2\delta}\overline{A}_x + (1-r)\times(\frac{\lambda}{2\delta+\lambda} + \frac{2\delta}{2\delta+\lambda}{}^{(2\delta+\lambda)}\overline{A}_x) - (r\times\overline{A}_x + (1-r)\times(\frac{\lambda}{\delta+\lambda} + \frac{\delta}{\delta+\lambda}{}^{(\delta+\lambda)}\overline{A}_x))^2$$

$$\overline{P}_x^* = \frac{\overline{A}_x^*}{\overline{a}_x^*} = \frac{\delta\overline{A}_x^*}{1-\overline{A}_x^*}$$

$$Var[L^*] = (1+\frac{\overline{P}_x^*}{\delta})^2\times Var[Z^*]$$

<Numerical example>

(4-13) Using the 2007 Japan standard mortality table for male, whole life single premium, present value of life annuities and whole life net premium rate are calculated and shown in Appendix A. The author assumed that λ=0.02、E[R]=r=0.98、i=2%. In calculation, uniform distribution of death assumption for fractional years is assumed and the formula, $\overline{A}_x = \frac{i}{\delta}A_x$, is used. For age, X=30, 50, 70, the result is shown in the following table.

<Table 1> Comparison of Net Premium Rate (Formula base)

Mortality Table: 2007 Japan Standard Mortality Table (Male)
Assumed Interest Rate: i=2%

Net Premium Rate

Age	30	50	70
Std Model	0.01269	0.02522	0.06383
Model A	0.01296	0.02548	0.06411
Model B	0.01289	0.02543	0.06408

Relative value (Std model =100%)

Age	30	50	70
Std Model	100.00%	100.00%	100.00%
Model A	102.18%	101.06%	100.43%
Model B	101.64%	100.86%	100.38%

Single Premium

Age	30	50	70
Std Model	0.39047	0.55061	0.76323
Model A	0.39562	0.55331	0.76400
Model B	0.39434	0.55281	0.76392

Relative value (Std model =100%)

Age	30	50	70
Std Model	100.00%	100.00%	100.00%
Model A	101.32%	100.49%	100.10%
Model B	100.99%	100.40%	100.09%

For X=30 male, the author also performed simulation with N=21,000 and simulation trial 10,800 times to find simulated results, to confirm the theoretical figures. As shown below, the theoretical calculation has very close result to the simulated result, as expected.

<Table 2> Comparison of Net Premium Rate (Formula vs. Simulation)

Age 30

	Standard Model			Model A			Model B		
	Formula(1)	Sim(2)	(2)/(1)	Formula(1)	Sim(2)	(2)/(1)	Formula(1)	Sim(2)	(2)/(1)
Single Premium	0.39047	0.39046	99.998%	0.39562	0.39561	99.998%	0.39434	0.39434	99.999%
Net Premium Rate	0.01269	0.01269	99.997%	0.01296	0.01296	99.997%	0.01289	0.01289	99.998%

Additional net premium with extra mortality by catastrophic event, expressed in relative to that for standard model does not look significant. Premium based on model A is 102.18% of the premium based on standard model. Such small increase is, in reality, might be covered by medical selection. As we see later; however, that the mortality risk measured by standard deviation is much larger in model A or B, compared to the standard model.

<Non-diversifiable mortality risk of insurance portfolio>

(4-14) Next we examine an insurance portfolio of n contacts for model A and model B. We will see that the mortality risk is not diversifiable in this portfolio. As in (4-5), we assume that n policies of continuous pay whole life are issued at the same time (t=0) for all age (x). Insurance amount is 1. All insured follow the same mortality assumption (including catastrophic) and death occurs randomly. Termination other than death is not assumed. We examine the expected value of future loss as follows.

(4-15) First we assume model A. In this case, insurance event (death by the insured) occurs randomly based on mortality table and the occurrence of the catastrophic event. The insurance event; however, is not independent, since catastrophic event affects all the insured similarly and at the same time. There is a correlation between the deaths by the insured.

(4-16) For an insurance portfolio with n insured, let Z^*_k and L^*_k be present value of benefit payment and present value of future loss for k th insured (k =1,2,・・・ n ） . Let the sum of L^*_k be $^{Port}L^*$. We have the following formula,

$$L^*_k = \frac{(\delta + \overline{P}^*_x)Z^*_k - \overline{P}^*_x}{\delta}$$

$$Var[L^*_k] = (1 + \frac{\overline{P}^*_x}{\delta})^2 Var[Z^*]$$

$$Cov[L^*_k, L^*_l] = (1 + \frac{\overline{P}^*_x}{\delta})^2 Cov[Z^*_k, Z^*_l] \ (k \neq l).$$

$$^{Port}L^* = L^*_1 + L^*_2 + \Lambda \ L^*_n$$

$$Var[^{Port}L^*] = nVar[L^*] + \sum_{k \neq l} Cov[L^*_k, L^*_l]$$

We can calculate $Var[^{Port}L^*]$ experimentally by developing a simulation model, as shown later. Also if we can calculate either $Cov[L^*_k, L^*_l]$ or $Cov[Z^*_k, Z^*_l]$ in some way, we can calculate $Var[^{Port}L^*]$ in a theoretical way, using spreadsheets. In order to do that, we will determine joint cumulative probability distribution function of two lifetime stochastic variables as follows.

Let T^*_1, T^*_2, …T^*_n be n lifetime stochastic variables of insured aged (x) , a joint probability density function of two of these variables, T^*_1、T^*_2 is expressed as,

$$F_{T_1^*, T_2^*}(t_1, t_2) = \Pr[T_1^* \le t_1, T_2^* \le t_2]$$

First we assume $t_1 \le t_2$. If there are U catastrophic event happened in time 0 to time t_1 , and there are V catastrophic event happened in time t_1 and t_2, U and V follow Poisson distribution with average λt_1 and $\lambda(t_2 - t_1)$ respectively, and we have,

$$\begin{aligned}
F_{T_1^*, T_2^*}(t_1, t_2) &= \sum_{u=0}^{\infty}\sum_{v=0}^{\infty}(1 - r^u {}_{t_1}p_x)(1 - r^{u+v} {}_{t_2}p_x) \times e^{-\lambda t_1}\frac{(\lambda t_1)^u}{u!} \times e^{-\lambda(t_2 - t_1)}\frac{(\lambda(t_2 - t_1))^v}{v!} \\
&= 1 - {}_{t_1}p_x \sum_{u=0}^{\infty} e^{-\lambda t_1}\frac{(\lambda r t_1)^u}{u!} - {}_{t_2}p_x \sum_{u=0}^{\infty} e^{-\lambda t_1}\frac{(\lambda r t_1)^u}{u!}\sum_{v=0}^{\infty} e^{-\lambda(t_2 - t_1)}\frac{(\lambda r(t_2 - t_1))^v}{v!} \\
&\quad + {}_{t_1}p_x \, {}_{t_2}p_x \sum_{u=0}^{\infty} e^{-\lambda t_1}\frac{(\lambda r^2 t_1)^u}{u!}\sum_{v=0}^{\infty} e^{-\lambda(t_2 - t_1)}\frac{(\lambda r(t_2 - t_1))^v}{v!} \\
&= 1 - {}_{t_1}p_x e^{-\lambda(1-r)t_1} - {}_{t_2}p_x e^{-\lambda(1-r)t_2} + {}_{t_1}p_x \, {}_{t_2}p_x \exp(-\lambda t_1 + \lambda r^2 t_1 - \lambda(t_2 - t_1) + \lambda r(t_2 - t_1)) \\
&= (1 - {}_{t_1}p_x e^{-\lambda(1-r)t_1})(1 - {}_{t_2}p_x e^{-\lambda(1-r)t_2}) + {}_{t_1}p_x \, {}_{t_2}p_x e^{-\lambda(1-r)(t_1 + t_2)}(e^{t_1\lambda(1-r)^2} - 1)
\end{aligned}$$

Also consider the situation of $t_1 > t_2$ and consider symmetric property, we have,

$$F_{T_1^*, T_2^*}(t_1, t_2) = (1 - {}_{t_1}p_x e^{-\lambda(1-r)t_1})(1 - {}_{t_2}p_x e^{-\lambda(1-r)t_2}) + {}_{t_1}p_x \, {}_{t_2}p_x e^{-\lambda(1-r)(t_1 + t_2)}(e^{\lambda Min(t_1, t_2)(1-r)^2} - 1).$$

The second term implies the correlation between T^*_1 と T^*_2, and we confirmed that that T^*_1 と T^*_2 are not independent.

In addition, above formula is also valid if we assume that the age of the first insured is (x) and that of the second (y), where (x) and (y) might be different. Therefore, we have

$$F_{T_1^*,T_2^*}(t_1,t_2) = (1-{}_{t_1}p_x e^{-\lambda(1-r)t_1})(1-{}_{t_2}p_y e^{-\lambda(1-r)t_2})+{}_{t_1}p_x\,{}_{t_2}p_y e^{-\lambda(1-r)(t_1+t_2)}(e^{\lambda Min(t_1,t_2)(1-r)^2}-1).$$

(4-17) Once we have joint cumulative probability distribution function, we might calculate the joint probability density function and can calculate the covariance of Z_1,Z_2. Although joint cumulative probability distribution function is continuous but not differentiable when $t_1=t_2$, following alternative approach can be taken. Discrete version of the joint probability density function can be defined for m_1=1,2,⋯、m_2=1,2,⋯,

$$\begin{aligned} f_{T_1^*,T_2^*}(m_1,m_2) &= \Pr(m_1-1 \le T_1^* < m_1, m_2-1 \le T_2^* < m_2) \\ &= F_{T_1^*,T_2^*}(m_1,m_2) + F_{T_1^*,T_2^*}(m_1-1,m_2-1) - F_{T_1^*,T_2^*}(m_1-1,m_2) - F_{T_1^*,T_2^*}(m_1,m_2-1) \end{aligned}$$

Using this formula and assuming the uniform deaths occurs at the fraction of years, we have,

$$Cov[Z^*{}_1, Z^*{}_2] = (\frac{i}{\delta})^2 \sum_{m_1=1}^{\infty} \sum_{m_2=1}^{\infty} e^{-\delta(m_1+m_2)} f_{T_1^*,T_2^*}(m_1,m_2),$$

which can be used to calculate the covariance using spreadsheets.

(4-18) Using the covariance calculated as above, we can calculate the variance of the insurance portfolio defined in (4-16),

$$Var[{}^{Port}L^*] = (1+\frac{\overline{P}_x^*}{\delta})^2 (nVar[Z^*] + n(n-1)Cov[Z^*{}_1, Z^*{}_2])$$

$$Var[\frac{{}^{Port}L^*}{n}] = (1+\frac{\overline{P}_x^*}{\delta})^2 (\frac{1}{n}Var[Z^*] + (1-\frac{1}{n})Cov[Z^*{}_1, Z^*{}_2])$$

Above expression tells that the variance of portfolio loss per contract does not converge to zero for significantly large n. In other words, Portfolio loss is not diversifiable.

For large n, we have an approximation,

$$Var[\frac{{}^{Port}L^*}{n}] \approx (1+\frac{\overline{P}_x^*}{\delta})^2 Cov[Z^*{}_1, Z^*{}_2].$$

If divided by premium rate, we have the following expression.

$$\begin{aligned} \frac{1}{\overline{P}_x^*} Std[\frac{{}^{Port}L^*}{n}] &\approx (\frac{1}{\delta \overline{A}_x^*})\sqrt{Cov[Z^*{}_1, Z^*{}_2]} \\ &= (\frac{1}{\delta})\sqrt{\frac{Cov[Z^*{}_1, Z^*{}_2]}{E[Z^*{}_1]E[Z^*{}_2]}} = (\frac{1}{\delta})\sqrt{Corr[Z^*{}_1, Z^*{}_2]} \end{aligned}$$

(4-19) Also for model B, we can calculate joint cumulative probability distribution function in a closed form and we can calculate covariance. Let E[R]=r, and Var[R]=r. Let the probability density function of R be $g_R(s)$. Assume that $t_1 \le t_2$. We have the following formula, by considering when the catastrophe occurs either before t_1, sometime between t_1 and t_2, or after t_2,

$$
\begin{aligned}
&F_{T_1^*,T_2^*}(t_1,t_2)\\
&=\int_0^1 g_R(s)((1-s_{t_1}p_x)(1-s_{t_2}p_x)(1-e^{-\lambda t_1})+(1-_{t_1}p_x)(1-s_{t_2}p_x)(e^{-\lambda t_1}-e^{-\lambda t_2})+(1-_{t_1}p_x)(1-_{t_2}p_x)(e^{-\lambda t_2}))\\
&=\int_0^1 g_R(s)((1-s_{t_1}p_x)(1-s_{t_2}p_x)-(1-s)({}_{t_1}p_x e^{-\lambda t_1}+{}_{t_2}p_x e^{-\lambda t_2})+(1-s)_{t_1}p_{x\,t_2}p_x(se^{-\lambda t_1}+e^{-\lambda t_2}))\\
&=(1-r_{t_1}p_x)(1-r_{t_2}p_x)+\sigma^2{}_{t_1}p_{x\,t_2}p_x-(1-r)({}_{t_1}p_x e^{-\lambda t_1}+{}_{t_2}p_x e^{-\lambda t_2})\\
&\qquad +{}_{t_1}p_{x\,t_2}p_x((r-r^2-\sigma^2)e^{-\lambda t_1}+(1-r)e^{-\lambda t_2})\\
&=(1-r_{t_1}p_x)(1-r_{t_2}p_x)-(1-r)({}_{t_1}p_x e^{-\lambda t_1}+{}_{t_2}p_x e^{-\lambda t_2})+(1-r)_{t_1}p_{x\,t_2}p_x(re^{-\lambda t_1}+e^{-\lambda t_2})\\
&\qquad +\sigma^2{}_{t_1}p_{x\,t_2}p_x(1-e^{-\lambda t_1})
\end{aligned}
$$

Also considering the situation that $t_1 < t_2$, and the case if the age of the second insured is (y), we have,

$$
\begin{aligned}
&F_{T_1^*,T_2^*}(t_1,t_2)\\
&=(1-r_{t_1}p_x)(1-r_{t_2}p_y)-(1-r)({}_{t_1}p_x e^{-\lambda t_1}+{}_{t_2}p_y e^{-\lambda t_2})+(1-r)_{t_1}p_{x\,t_2}p_y(re^{-\lambda Min(t_1,t_2)}+e^{-\lambda Max(t_1,t_2)})\\
&\qquad +\sigma^2{}_{t_1}p_{x\,t_2}p_y(1-e^{-\lambda Min(t_1,t_2)})
\end{aligned}
$$

Above expression tells that the joint cumulative probability distribution function reflects expected value and variance of R. The distribution of R is not relevant. Also, if $\sigma^2=0$, the forth term becomes zero.

For model B, insurance risk caused by catastrophic event is not diversifiable, using the same argument as in (5-17) and (5-18).

(Chapter 5) Distribution of Present Value of Future Loss by Stochastic Simulation

(5-1) In the previous chapter, we define two models and get the closed expressions of some actuarial notation. Also, for insurance portfolio with n contracts, we get a theoretical value of the variance of present value of future loss by determining the joint cumulative probability distribution function and using it in the discrete version.

We know that insurance risk for catastrophic event is not diversifiable and we expect that actual distribution of future loss has a distribution with long tail in the left side. In order to obtain such distribution, we need to perform stochastic simulation. Following is an example of such stochastic simulation for model A and model B. Our approach can be used to check the validity of the simulation models. Simulation result is compared to the theoretical values calculated using above approach.

(5-2) Stochastic simulation assumption
- Mortality table: 2007 Japan standard mortality table (For fractional years, uniform deaths assumption is used)
- Assumed interest: i=2%
- An insurance portfolio of x=30, male, n=21,000, of continuous premium pay, immediate benefit pay whole life, all contracted simultaneously at time 0
- Termination other than death is not assumed.
- Insurance amount, 10 million yen per policy (= crude approximation, $100,000 per policy)
- Simulation 10,800 times
- Model parameters: as follows

<Table 3> Simulation Parameters

SIM1	Standard model	λ=0.00、R=r=1, i.e. standard life contingency model
SIM2	Model A	λ=0.02、R=r=0.98 (constant)
SIM3	Model B1	λ=0.02、R=r=0.98 (constant)
SIM4	Model B2	λ=0.02、R has the following probability density function $g_R(s)=49s^{48}$、E[R]=0.98

For simulation, the author assumes parameters as λ=0.02 and r=0.98, considering a possible worst case scenario. Also for model B2, power function is used for the distribution of R.

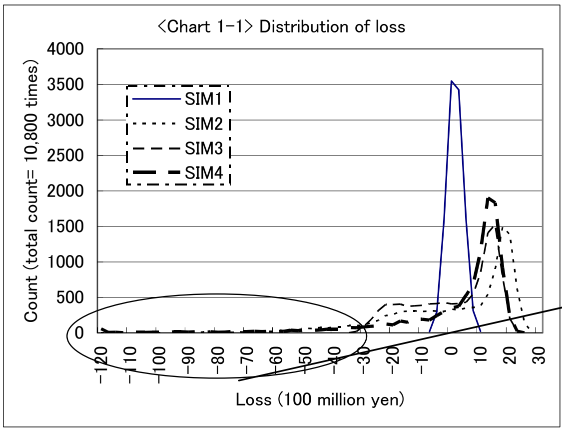
(5-3) Simulation result and observations
First, a simulation result and the comparison with theoretical calculation are shown as Table 4. For a distribution of each simulation, readers may see Appendix B. As expected, variance of the insurance portfolio is close to each other. The difference is in the 1% range, and often much smaller than 1%.

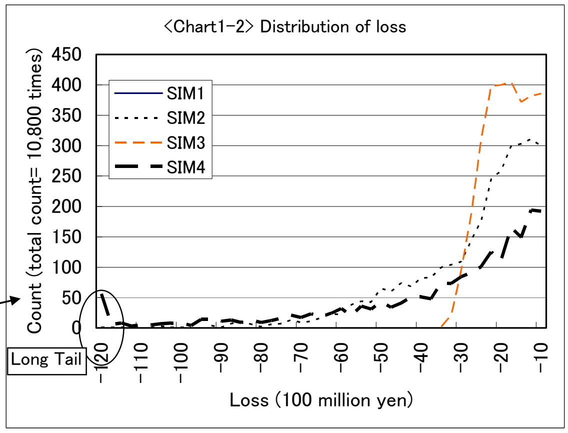
By stochastic simulation, we see that the portfolio risk has a distribution with long tail, as shown in Chart 1. Generally speaking, in order to quantify the mortality risk inherited in insurance portfolio, we may use several risk measures, such as standard deviation, value at risk (VaR) and tail value at risk (TVaR). As a risk measure, standard deviation has a shortage of not incorporating the asymmetric risk distribution or risk with long tail. But as we have shown, we can calculate standard deviation measure much easier than performing simulation. In Appendix C, standard deviation of insurance portfolio per policy for all x is shown, assuming the n is large enough. Such convenience can be utilized, for example, to perform sensitivity analysis.

<Table 4> Portfolio risk comparison (Formula bs Simulation)

	Symbol	(SIM1) Standard Model			(SIM2) Model A		
		Formula(1)	Sim(2)	(2)/(1)	Formula(1)	Sim(2)	(2)/(1)
N=21,000、Insured amount =1							
Expected value of loss	E[L]	0.00000	0.00000	–	0.00000	0.00000	–
Variance of loss	Var[L]	719.66392	711.03772	98.80%	45,290.30538	44,873.24145	99.08%
Standard deviation of loss	Std[L]	26.82655	26.66529	99.40%	212.81519	211.83305	99.54%
Variance of loss per policy	Var[L/N]	0.03427	0.03386	98.80%	2.15668	2.13682	99.08%
Standard deviation of loss per policy	Std[L/N]	0.00128	0.00127	99.40%	0.01013	0.01009	99.54%
N=21,000、Insured amount =10 mil yen							
Expected value of loss	E[L]	0	0	–	0	0	–
Variance of loss	Var[L]	7.19664E+16	7.11038E+16	98.80%	4.52903E+18	4.48732E+18	99.08%
Standard deviation of loss	Std[L]	268,265,525	266,652,905	99.40%	2,128,151,907	2,118,330,509	99.54%
Variance of loss per policy	Var[L/N]	3.42697E+12	3.38589E+12	98.80%	2.15668E+14	2.13682E+14	99.08%
Standard deviation of loss per policy	Std[L/N]	12,775	12,698	99.40%	101,341	100,873	99.54%

	Symbol	(SIM2) Model B1			(SIM2) Model B2		
		Formula(1)	Sim(2)	(2)/(1)	Formula(1)	Sim(2)	(2)/(1)
N=21,000、Insured amount =1							
Expected value of loss	E[L]	0.00000	0.00000	–	0.00000	0.00000	–
Variance of loss	Var[L]	19,950.06964	19,929.35301	99.90%	55,641.62648	55,635.59084	99.99%
Standard deviation of loss	Std[L]	141.24472	141.17136	99.95%	235.88477	235.87198	99.99%
Variance of loss per policy	Var[L/N]	0.95000	0.94902	99.90%	2.64960	2.64931	99.99%
Standard deviation of loss per policy	Std[L/N]	0.00673	0.00672	99.95%	0.01123	0.01123	99.99%
N=21,000、Insured amount =10 mil yen							
Expected value of loss	E[L]	0	0	–	0	0	–
Variance of loss	Var[L]	1.99501E+18	1.99294E+18	99.90%	5.56416E+18	5.56356E+18	99.99%
Standard deviation of loss	Std[L]	1,412,447,154	1,411,713,605	99.95%	2,358,847,737	2,358,719,798	99.99%
Variance of loss per policy	Var[L/N]	9.50003E+13	9.49017E+13	99.90%	2.6496E+14	2.64931E+14	99.99%
Standard deviation of loss per policy	Std[L/N]	67,259	67,224	99.95%	112,326	112,320	99.99%





<Table 5> Standard deviation of loss per policy, divided by net premium rate

	Symbol	Std Model	Model A1	Model B1	Model B2
Standard deviation of loss per policy	(1/P˜x)Std[L/N]	0.101	0.782	0.522	0.871

In order to gasp the level of risk, standard deviation per policy might be standardized by, for example, dividing it with net premium rate. Table 5 shows the standard deviation divided by net premium rate. We see that the standard deviation of loss in the standard model is about 0.1 years of premium, while that in the model B2 is 0.87 years of premium. Under model B2, if the company want to cover the loss with the 99.3% of probability, the company needs to have 2.5 times of standard deviation, or in this case, 2.18 (0.871 * 2.5) years of net premium as surplus.

(Chapter 6) Conclusion

(6-1) Even though we are not certain the timing, it is a well formed opinion among the authorities that influenza pandemic is an incident that occurs periodically in every several decades. Once it happens, its impact to our human lives might be huge and severe. Pandemic risk is a kind of catastrophic risk. The research for the quantitative evaluation of pandemic risk has just been started.

There are several important differences between the natures of pandemic risk compared to the earthquake risk. The author understands that the most sophisticated risk evaluation model should reflect the nature of the pandemic risk. As a starting point, however, the author found that some simple assumptions about the catastrophic event, which can be applied both for pandemic risk as well as earthquake, is useful to apply it for our traditional, standard life contingency model to extend.

In this paper, the author extended the standard life contingency model with an additional mortality assumption caused by pandemic or other catastrophic event and showed some closed expression of actuarial notations such as single premium, net premium rate for continuous premium pay whole life. Further, the author considered an insurance portfolio and its mortality risk and showed a theoretical approach to calculate the standard deviation of the loss stochastic variable (L) of that portfolio, without using stochastic simulation. The standard deviation can be used as a risk measure like value at risk (VaR) or tail value at risk (TVaR). The key idea in this paper's approach is to express the joint cumulative probability distribution function of the lifetime stochastic variables of two insured in a closed form and use to derive the discrete version of joint probability density function.

In order to do some more detailed analysis regarding the distribution of the loss stochastic variable, stochastic simulation is a strong approach. One advantage of our approach is its convenience. Our approach can compliment the stochastic simulation approach.

In this paper, the author developed a prototype simulation model which does not include the nature of the pandemic risk. In order to incorporate such characteristics, the author think we need to have deeper understanding about the mathematics behind infectious diseases and epidemic modeling. For example, basic reproductive ratio, SIR (Susceptible, Infective and Removed) model and agency based modeling are some key ideas, which might be an interesting area for an actuary.

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<Appendix A> Theoretical Calculation Result of Single Premium and Some Other Actuarial Figures

Mortality Table: 2007 Japan Mortality Table (Male

Asuumed Interest i= 2.00%

Parameters λ= 0.02 r= 0.98

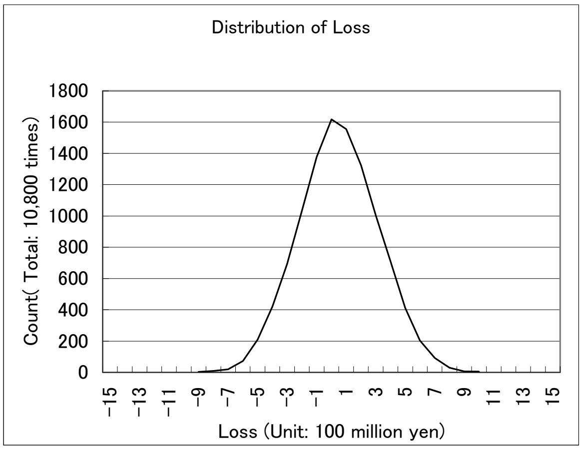
<Note> Uniform distribution of deaths in each year of age is assumed when converting discrete model into continuing model

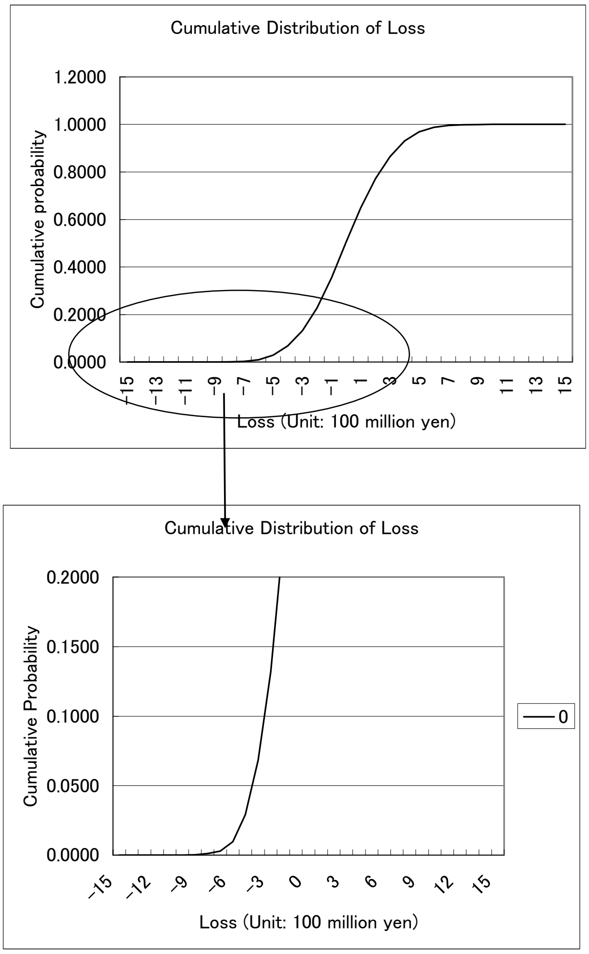
Age	Mortality	Standard Model (Discrete)			Standard Model (Continious)			Model A			Model B		
x	q_x	A_x	a due x	P_x	$\tilde{A}_x$	$\tilde{a}_x$	$\tilde{P}_x$	$\tilde{A}_x^{(*A)}$	$\tilde{a}_x^{(*A)}$	$\tilde{P}_x^{(*A)}$	$\tilde{A}_x^{(*B)}$	$\tilde{a}_x^{(*B)}$	$\tilde{P}_x^{(*B)}$
0	0.00108	0.22117	39.72016	0.00557	0.22338	39.21814	0.00570	0.23249	38.75799	0.00600	0.22953	38.90754	0.00590
1	0.00075	0.22476	39.53726	0.00568	0.22700	39.03523	0.00582	0.23599	38.58101	0.00612	0.23309	38.72765	0.00602
2	0.00049	0.22868	39.33751	0.00581	0.23096	38.83548	0.00595	0.23983	38.38737	0.00625	0.23698	38.53106	0.00615
3	0.00031	0.23287	39.12343	0.00595	0.23520	38.62139	0.00609	0.24394	38.17957	0.00639	0.24116	38.32026	0.00629
4	0.00021	0.23729	38.89796	0.00610	0.23966	38.39591	0.00624	0.24828	37.96050	0.00654	0.24556	38.09818	0.00645
5	0.00017	0.24188	38.66403	0.00626	0.24429	38.16198	0.00640	0.25279	37.73306	0.00670	0.25012	37.86771	0.00661
6	0.00016	0.24659	38.42385	0.00642	0.24905	37.92179	0.00657	0.25741	37.49941	0.00686	0.25481	37.63104	0.00677
7	0.00016	0.25140	38.17843	0.00658	0.25391	37.67636	0.00674	0.26214	37.26057	0.00704	0.25960	37.38919	0.00694
8	0.00016	0.25631	37.92807	0.00676	0.25887	37.42599	0.00692	0.26697	37.01682	0.00721	0.26448	37.14244	0.00712
9	0.00015	0.26132	37.67266	0.00694	0.26392	37.17057	0.00710	0.27190	36.76806	0.00739	0.26947	36.89068	0.00730
10	0.00014	0.26644	37.41172	0.00712	0.26909	36.90963	0.00729	0.27693	36.51381	0.00758	0.27456	36.63345	0.00749
11	0.00013	0.27166	37.14516	0.00731	0.27437	36.64306	0.00749	0.28208	36.25396	0.00778	0.27977	36.37063	0.00769
12	0.00014	0.27700	36.87285	0.00751	0.27976	36.37074	0.00769	0.28734	35.98841	0.00798	0.28508	36.10211	0.00790
13	0.00018	0.28244	36.59544	0.00772	0.28526	36.09331	0.00790	0.29269	35.71776	0.00819	0.29050	35.82852	0.00811
14	0.00025	0.28796	36.31388	0.00793	0.29083	35.81175	0.00812	0.29814	35.44299	0.00841	0.29600	35.55083	0.00833
15	0.00036	0.29355	36.02917	0.00815	0.29647	35.52703	0.00834	0.30364	35.16506	0.00863	0.30156	35.27000	0.00855
16	0.00049	0.29916	35.74262	0.00837	0.30215	35.24047	0.00857	0.30918	34.88529	0.00886	0.30716	34.98735	0.00878
17	0.00062	0.30481	35.45484	0.00860	0.30785	34.95268	0.00881	0.31474	34.60426	0.00910	0.31278	34.70349	0.00901
18	0.00073	0.31048	35.16574	0.00883	0.31357	34.66357	0.00905	0.32034	34.32189	0.00933	0.31843	34.41831	0.00925
19	0.00080	0.31619	34.87451	0.00907	0.31934	34.37234	0.00929	0.32597	34.03738	0.00958	0.32412	34.13102	0.00950
20	0.00084	0.32197	34.57967	0.00931	0.32518	34.07748	0.00954	0.33168	33.74925	0.00983	0.32988	33.84013	0.00975
21	0.00086	0.32784	34.28006	0.00956	0.33111	33.77786	0.00980	0.33748	33.45636	0.01009	0.33573	33.54452	0.01001
22	0.00085	0.33383	33.97488	0.00983	0.33715	33.47267	0.01007	0.34339	33.15792	0.01036	0.34169	33.24337	0.01028
23	0.00084	0.33994	33.66299	0.01010	0.34333	33.16077	0.01035	0.34943	32.85279	0.01064	0.34779	32.93556	0.01056
24	0.00083	0.34619	33.34426	0.01038	0.34964	32.84203	0.01065	0.35561	32.54084	0.01093	0.35402	32.62095	0.01085
25	0.00082	0.35258	33.01855	0.01068	0.35609	32.51631	0.01095	0.36192	32.22193	0.01123	0.36039	32.29941	0.01116
26	0.00081	0.35910	32.68572	0.01099	0.36268	32.18347	0.01127	0.36838	31.89592	0.01155	0.36689	31.97078	0.01148
27	0.00080	0.36577	32.34563	0.01131	0.36942	31.84337	0.01160	0.37498	31.56266	0.01188	0.37355	31.63494	0.01181
28	0.00081	0.37259	31.99815	0.01164	0.37630	31.49587	0.01195	0.38172	31.22200	0.01223	0.38034	31.29173	0.01215
29	0.00083	0.37953	31.64374	0.01199	0.38332	31.14146	0.01231	0.38861	30.87443	0.01259	0.38727	30.94164	0.01252
30	0.00086	0.38662	31.28258	0.01236	0.39047	30.78028	0.01269	0.39562	30.52010	0.01296	0.39434	30.58482	0.01289
31	0.00089	0.39383	30.91482	0.01274	0.39775	30.41251	0.01308	0.40277	30.15915	0.01335	0.40154	30.22143	0.01329
32	0.00092	0.40117	30.54029	0.01314	0.40517	30.03798	0.01349	0.41005	29.79144	0.01376	0.40887	29.85129	0.01370
33	0.00096	0.40865	30.15885	0.01355	0.41272	29.65652	0.01392	0.41747	29.41678	0.01419	0.41633	29.47426	0.01413
34	0.00100	0.41626	29.77060	0.01398	0.42041	29.26826	0.01436	0.42502	29.03531	0.01464	0.42393	29.09045	0.01457
35	0.00105	0.42401	29.37539	0.01443	0.42824	28.87303	0.01483	0.43272	28.64684	0.01511	0.43167	28.69969	0.01504
36	0.00112	0.43190	28.97332	0.01491	0.43620	28.47095	0.01532	0.44055	28.25149	0.01559	0.43954	28.30209	0.01553
37	0.00119	0.43991	28.56478	0.01540	0.44429	28.06240	0.01583	0.44850	27.84964	0.01610	0.44755	27.89802	0.01604
38	0.00128	0.44805	28.14957	0.01592	0.45251	27.64718	0.01637	0.45659	27.44108	0.01664	0.45568	27.48730	0.01658
39	0.00137	0.45631	27.72806	0.01646	0.46086	27.22565	0.01693	0.46481	27.02617	0.01720	0.46394	27.07026	0.01714
40	0.00148	0.46471	27.30002	0.01702	0.46934	26.79759	0.01751	0.47316	26.60469	0.01778	0.47233	26.64671	0.01773
41	0.00161	0.47322	26.86578	0.01761	0.47794	26.36334	0.01813	0.48163	26.17695	0.01840	0.48084	26.21696	0.01834
42	0.00176	0.48185	26.42564	0.01823	0.48665	25.92319	0.01877	0.49022	25.74326	0.01904	0.48946	25.78129	0.01899
43	0.00192	0.49059	25.97988	0.01888	0.49548	25.47741	0.01945	0.49892	25.30388	0.01972	0.49820	25.33999	0.01966
44	0.00211	0.49944	25.52849	0.01956	0.50442	25.02601	0.02016	0.50773	24.85881	0.02042	0.50705	24.89305	0.02037
45	0.00231	0.50839	25.07196	0.02028	0.51346	24.56947	0.02090	0.51665	24.40853	0.02117	0.51601	24.44094	0.02111
46	0.00254	0.51745	24.61025	0.02103	0.52260	24.10774	0.02168	0.52567	23.95299	0.02195	0.52506	23.98363	0.02189
47	0.00277	0.52659	24.14378	0.02181	0.53184	23.64125	0.02250	0.53478	23.49260	0.02276	0.53421	23.52154	0.02271
48	0.00304	0.53584	23.67223	0.02264	0.54118	23.16969	0.02336	0.54400	23.02705	0.02362	0.54346	23.05433	0.02357
49	0.00333	0.54517	23.19619	0.02350	0.55061	22.69363	0.02426	0.55331	22.55692	0.02453	0.55281	22.58259	0.02448
50	0.00365	0.55459	22.71576	0.02441	0.56012	22.21318	0.02522	0.56271	22.08230	0.02548	0.56223	22.10643	0.02543
51	0.00401	0.56409	22.23122	0.02537	0.56972	21.72863	0.02622	0.57219	21.60348	0.02649	0.57175	21.62611	0.02644
52	0.00440	0.57367	21.74303	0.02638	0.57938	21.24042	0.02728	0.58175	21.12091	0.02754	0.58133	21.14211	0.02750
53	0.00480	0.58331	21.25140	0.02745	0.58912	20.74878	0.02839	0.59138	20.63479	0.02866	0.59098	20.65460	0.02861
54	0.00522	0.59302	20.75605	0.02857	0.59893	20.25342	0.02957	0.60108	20.14484	0.02984	0.60071	20.16333	0.02979
55	0.00567	0.60281	20.25692	0.02976	0.60881	19.75426	0.03082	0.61086	19.65100	0.03109	0.61052	19.66821	0.03104
56	0.00615	0.61267	19.75406	0.03101	0.61877	19.25139	0.03214	0.62071	19.15332	0.03241	0.62040	19.16932	0.03236
57	0.00666	0.62260	19.24751	0.03235	0.62880	18.74483	0.03355	0.63064	18.65184	0.03381	0.63035	18.66667	0.03377
58	0.00718	0.63260	18.73725	0.03376	0.63891	18.23455	0.03504	0.64065	18.14653	0.03530	0.64038	18.16025	0.03526
59	0.00774	0.64269	18.22284	0.03527	0.64910	17.72012	0.03663	0.65074	17.63694	0.03690	0.65049	17.64960	0.03686

Age	Mortality	Standard Model (Discrete)			Standard Model (Continuous)			Model A			Model B		
x	q_x	A_x	a due x	P_x	$\tilde{A}_x$	$\tilde{a}_x$	$\tilde{P}_x$	$\tilde{A}_x^{(*A)}$	$\tilde{a}_x^{(*A)}$	$\tilde{P}_x^{(*A)}$	$\tilde{A}_x^{(*B)}$	$\tilde{a}_x^{(*B)}$	$\tilde{P}_x^{(*B)}$
60	0.00834	0.65286	17.70433	0.03688	0.65936	17.20159	0.03833	0.66092	17.12314	0.03860	0.66069	17.13480	0.03856
61	0.00902	0.66310	17.18171	0.03859	0.66971	16.67895	0.04015	0.67118	16.60510	0.04042	0.67096	16.61581	0.04038
62	0.00981	0.67342	16.65558	0.04043	0.68013	16.15281	0.04211	0.68151	16.08342	0.04237	0.68131	16.09323	0.04234
63	0.01072	0.68379	16.12689	0.04240	0.69060	15.62410	0.04420	0.69189	15.55906	0.04447	0.69171	15.56801	0.04443
64	0.01180	0.69418	15.59663	0.04451	0.70110	15.09382	0.04645	0.70231	15.03297	0.04672	0.70215	15.04112	0.04668
65	0.01306	0.70458	15.06634	0.04677	0.71160	14.56352	0.04886	0.71273	14.50671	0.04913	0.71258	14.51411	0.04910
66	0.01452	0.71495	14.53753	0.04918	0.72208	14.03469	0.05145	0.72312	13.98177	0.05172	0.72299	13.98847	0.05168
67	0.01616	0.72526	14.01173	0.05176	0.73249	13.50887	0.05422	0.73346	13.45969	0.05449	0.73334	13.46573	0.05446
68	0.01794	0.73549	13.48996	0.05452	0.74282	12.98709	0.05720	0.74372	12.94147	0.05747	0.74362	12.94691	0.05744
69	0.01986	0.74564	12.97249	0.05748	0.75307	12.46960	0.06039	0.75390	12.42739	0.06066	0.75381	12.43226	0.06063
70	0.02193	0.75570	12.45938	0.06065	0.76323	11.95647	0.06383	0.76400	11.91752	0.06411	0.76392	11.92187	0.06408
71	0.02415	0.76567	11.95065	0.06407	0.77331	11.44772	0.06755	0.77402	11.41187	0.06783	0.77394	11.41574	0.06780
72	0.02657	0.77557	11.44608	0.06776	0.78330	10.94314	0.07158	0.78395	10.91023	0.07185	0.78388	10.91367	0.07183
73	0.02923	0.78538	10.94584	0.07175	0.79320	10.44288	0.07596	0.79380	10.41276	0.07623	0.79374	10.41579	0.07621
74	0.03223	0.79509	10.45021	0.07608	0.80302	9.94724	0.08073	0.80356	9.91976	0.08101	0.80351	9.92243	0.08098
75	0.03568	0.80470	9.96024	0.08079	0.81272	9.45724	0.08594	0.81322	9.43226	0.08622	0.81317	9.43460	0.08619
76	0.03961	0.81416	9.47760	0.08590	0.82228	8.97459	0.09162	0.82273	8.95195	0.09190	0.82269	8.95399	0.09188
77	0.04400	0.82346	9.00379	0.09146	0.83166	8.50077	0.09783	0.83207	8.48032	0.09812	0.83203	8.48208	0.09809
78	0.04877	0.83256	8.53961	0.09749	0.84085	8.03657	0.10463	0.84122	8.01816	0.10491	0.84119	8.01969	0.10489
79	0.05425	0.84148	8.08469	0.10408	0.84986	7.58164	0.11209	0.85019	7.56513	0.11238	0.85016	7.56644	0.11236
80	0.06039	0.85018	7.64091	0.11127	0.85865	7.13784	0.12030	0.85894	7.12309	0.12059	0.85892	7.12421	0.12056
81	0.06728	0.85865	7.20908	0.11911	0.86720	6.70600	0.12932	0.86746	6.69287	0.12961	0.86744	6.69382	0.12959
82	0.07500	0.86686	6.79010	0.12767	0.87550	6.28701	0.13926	0.87573	6.27536	0.13955	0.87572	6.27616	0.13953
83	0.08364	0.87481	6.38476	0.13702	0.88353	5.88165	0.15022	0.88373	5.87137	0.15052	0.88372	5.87204	0.15050
84	0.09329	0.88247	5.99378	0.14723	0.89127	5.49066	0.16232	0.89145	5.48160	0.16263	0.89144	5.48217	0.16261
85	0.10407	0.88985	5.61773	0.15840	0.89872	5.11460	0.17572	0.89887	5.10667	0.17602	0.89887	5.10713	0.17600
86	0.11609	0.89692	5.25720	0.17061	0.90586	4.75406	0.19054	0.90599	4.74713	0.19085	0.90599	4.74752	0.19083
87	0.12946	0.90367	4.91266	0.18395	0.91268	4.40950	0.20698	0.91280	4.40348	0.20729	0.91279	4.40380	0.20727
88	0.14432	0.91011	4.58441	0.19852	0.91918	4.08124	0.22522	0.91928	4.07603	0.22553	0.91928	4.07629	0.22552
89	0.16079	0.91622	4.27274	0.21443	0.92535	3.76956	0.24548	0.92544	3.76507	0.24580	0.92544	3.76528	0.24578
90	0.17900	0.92200	3.97778	0.23179	0.93119	3.47459	0.26800	0.93127	3.47073	0.26832	0.93127	3.47091	0.26831
91	0.19910	0.92746	3.69955	0.25070	0.93670	3.19636	0.29305	0.93677	3.19306	0.29338	0.93677	3.19320	0.29336
92	0.22119	0.93259	3.43806	0.27125	0.94188	2.93486	0.32093	0.94194	2.93205	0.32126	0.94194	2.93216	0.32124
93	0.24540	0.93739	3.19311	0.29357	0.94673	2.68989	0.35196	0.94678	2.68751	0.35229	0.94678	2.68760	0.35228
94	0.27184	0.94187	2.96444	0.31772	0.95126	2.46122	0.38650	0.95130	2.45921	0.38683	0.95130	2.45928	0.38682
95	0.30058	0.94604	2.75177	0.34379	0.95547	2.24855	0.42493	0.95551	2.24685	0.42527	0.95551	2.24690	0.42525
96	0.33166	0.94991	2.55470	0.37183	0.95938	2.05147	0.46765	0.95940	2.05004	0.46799	0.95940	2.05008	0.46798
97	0.36510	0.95348	2.37273	0.40185	0.96298	1.86949	0.51510	0.96300	1.86830	0.51544	0.96300	1.86834	0.51543
98	0.40085	0.95676	2.20537	0.43383	0.96629	1.70212	0.56770	0.96631	1.70113	0.56804	0.96631	1.70115	0.56803
99	0.43880	0.95976	2.05203	0.46771	0.96933	1.54878	0.62587	0.96935	1.54796	0.62621	0.96935	1.54798	0.62620
100	0.47877	0.96251	1.91211	0.50338	0.97210	1.40885	0.68999	0.97211	1.40817	0.69034	0.97211	1.40818	0.69033
101	0.52048	0.96500	1.78491	0.54064	0.97462	1.28165	0.76044	0.97463	1.28108	0.76079	0.97463	1.28110	0.76078
102	0.56359	0.96726	1.66961	0.57934	0.97690	1.16635	0.83758	0.97691	1.16588	0.83792	0.97691	1.16588	0.83792
103	0.60761	0.96931	1.56504	0.61935	0.97897	1.06178	0.92201	0.97898	1.06139	0.92236	0.97898	1.06140	0.92235
104	0.65200	0.97120	1.46880	0.66122	0.98088	0.96554	1.01589	0.98089	0.96522	1.01623	0.98089	0.96523	1.01622
105	0.69612	0.97306	1.37408	0.70815	0.98276	0.87081	1.12855	0.98276	0.87056	1.12888	0.98276	0.87057	1.12887
106	0.73925	0.97538	1.25564	0.77680	0.98510	0.75236	1.30935	0.98510	0.75219	1.30964	0.98510	0.75220	1.30964
107	1.00000	0.98039	1.00000	0.98039	0.99016	0.49672	1.99342	0.99017	0.49665	1.99369	0.99017	0.49665	1.99369

<Appendix B> Simulation Result

（SIM1） Standard Model, Simulation Result





Simulation Assumpions and Result

N：21,000

Insurance Amount：10 million yen

Simulaton：10,800 times

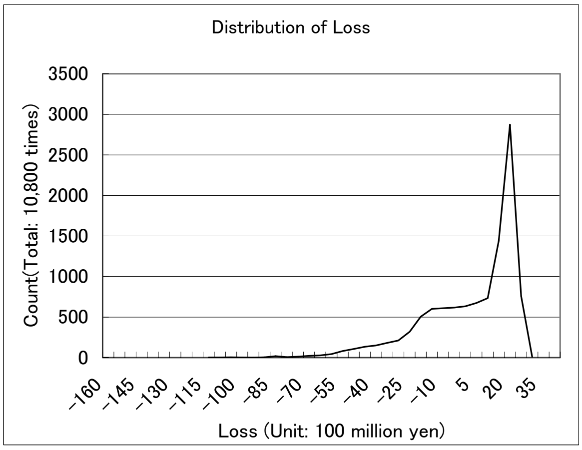
(Unit: yen)

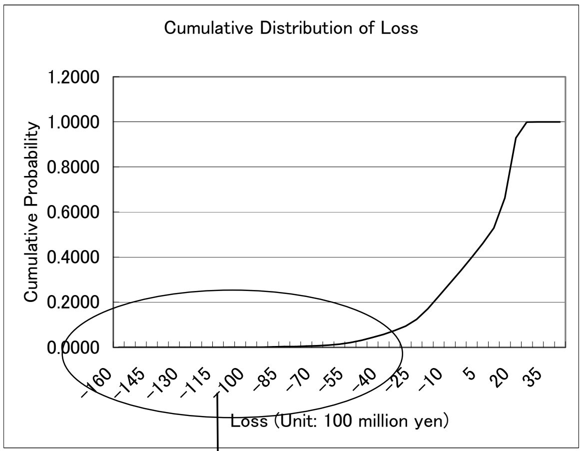
Maximum of L	997,205,623
Miinimum of L	−945,271,795
Average of L	−0
Median of L	−3,240,661
Std Deviation of L	266,652,905
Left 0.1% point	−818,872,070
Left 1% point	−597,844,627
Left 5% point	−434,962,284
Left 10% point	−340,204,438

Per policy, per insurance amount

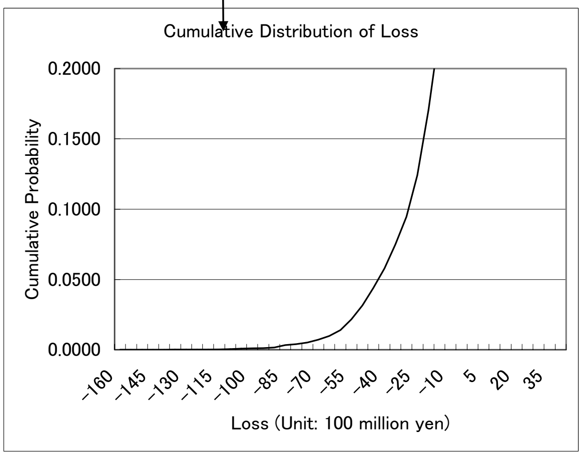
Maximum of L	0.00474860
Miinimum of L	−0.00450129
Average of L	−0.00000000
Median of L	−0.00001543
Std Deviation of L	0.00126978
Left 0.1% point	−0.00389939
Left 1% point	−0.00284688
Left 5% point	−0.00207125
Left 10% point	−0.00162002

（SIM2） Model A, Simulation Result





Simulation Assumptions and Result
N：21,000
Insurance Amount：10 million yen
Simulation：10,800 times



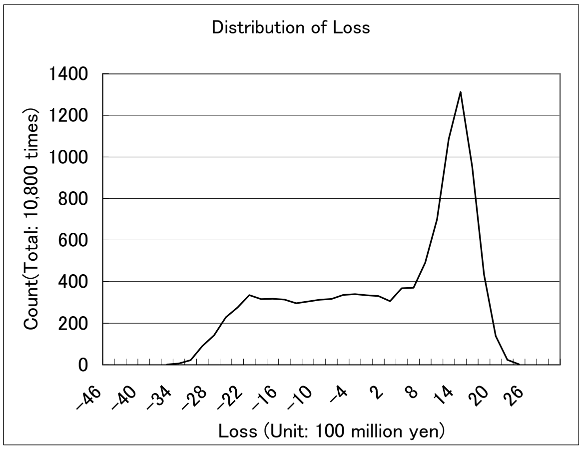
(Unit: yen)

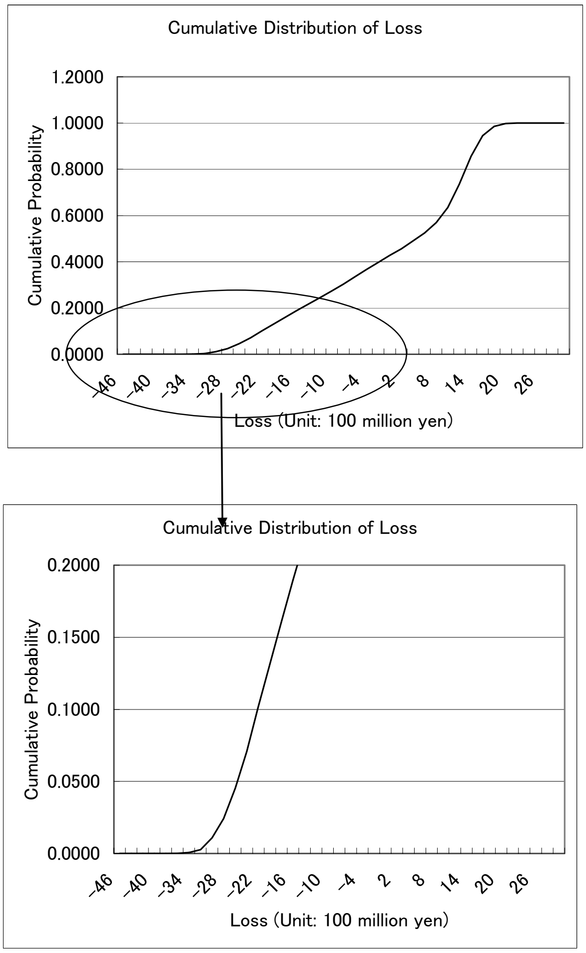
Maximum of L	2,742,172,806
Minimum of L	−13,812,581,352
Average of L	−0
Median of L	791,045,591
Std Deviation of L	2,118,330,509
Left 0.1% point	−10,159,553,993
Left 1% point	−6,492,448,616
Left 5% point	−4,258,699,583
Left 10% point	−2,887,088,157

Per policy, per insurance amount

Maximum of L	0.01305797
Minimum of L	−0.06577420
Average of L	−0.00000000
Median of L	0.00376688
Std Deviation of L	0.01008729
Left 0.1% point	−0.04837883
Left 1% point	−0.03091642
Left 5% point	−0.02027952
Left 10% point	−0.01374804

（SIM3） Model B1, Simulation Result





Simulation Assumptions and Result

N：21,000

Insurance Amount：10 million yen

Simulation：10,800 times

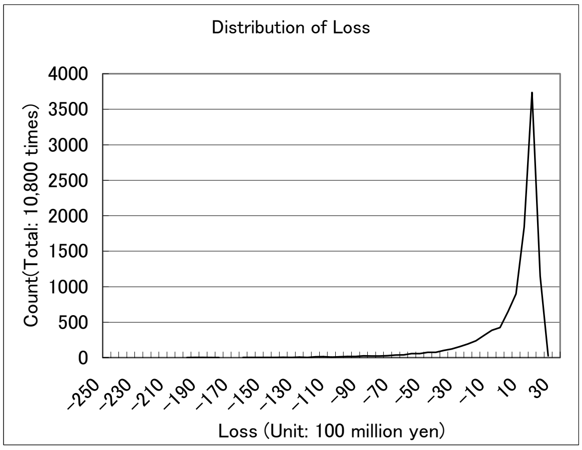
(Unit: yen)

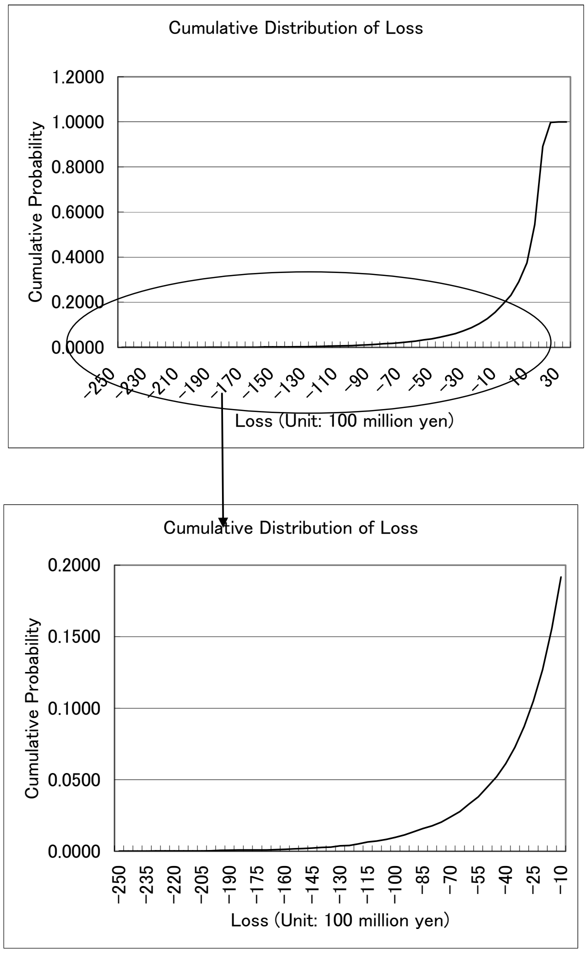
Maximum of L	2,293,351,849
Minimum of L	−3,670,193,204
Average of L	−0
Median of L	459,454,609
Std Deviation of L	1,411,713,605
Left 0.1% point	−3,320,663,071
Left 1% point	−3,024,352,806
Left 5% point	−2,558,994,237
Left 10% point	−2,210,155,318

Per policy, per insurance amount

Maximum of L	0.01092072
Minimum of L	−0.01747711
Average of L	−0.00000000
Median of L	0.00218788
Std Deviation of L	0.00672245
Left 0.1% point	−0.01581268
Left 1% point	−0.01440168
Left 5% point	−0.01218569
Left 10% point	−0.01052455

（SIM4） Model B, Simulation Result





Simulation Assumptions and Result

N : 21,000
Insurance Amount : 10 million yen
Simulation : 10,800 times

(Unit: yen)

Maximum of L	2,285,371,246
Minimum of L	−27,342,931,211
Average of L	−0
Median of L	906,498,429
Std Deviation of L	2,358,719,798
Left 0.1% point	−18,100,816,082
Left 1% point	−9,950,098,865
Left 5% point	−4,647,559,335
Left 10% point	−2,618,584,083

Per policy, per insurance amount

Maximum of L	0.01088272
Minimum of L	−0.13020443
Average of L	−0.00000000
Median of L	0.00431666
Std Deviation of L	0.01123200
Left 0.1% point	−0.08619436
Left 1% point	−0.04738142
Left 5% point	−0.02213123
Left 10% point	−0.01246945

<Appendix C> Standard Deviation of Loss of an Insurance Portfolio

Here we assume that n is large enough so that we can use the following approximation.

$$Std[\frac{{}^{Port}L^*}{n}] \approx (1+\frac{\overline{P}_x^*}{\delta})\sqrt{Cov[Z^*{}_1, Z^*{}_2]}$$

Right side value has calculated using the discrete version of probability density function in the paper

Age	Model A	Model B1	Model B2
0	0.01167	0.00655	0.01206
1	0.01163	0.00656	0.01204
2	0.01159	0.00656	0.01202
3	0.01154	0.00657	0.01200
4	0.01150	0.00657	0.01198
5	0.01145	0.00658	0.01195
6	0.01141	0.00658	0.01193
7	0.01136	0.00659	0.01191
8	0.01131	0.00659	0.01188
9	0.01126	0.00660	0.01186
10	0.01121	0.00660	0.01183
11	0.01116	0.00660	0.01181
12	0.01111	0.00661	0.01178
13	0.01106	0.00661	0.01175
14	0.01101	0.00661	0.01173
15	0.01096	0.00662	0.01170
16	0.01090	0.00662	0.01167
17	0.01085	0.00662	0.01164
18	0.01079	0.00662	0.01161
19	0.01074	0.00662	0.01157
20	0.01068	0.00662	0.01154
21	0.01062	0.00662	0.01151
22	0.01056	0.00662	0.01147
23	0.01050	0.00662	0.01144
24	0.01044	0.00662	0.01140
25	0.01037	0.00661	0.01136
26	0.01031	0.00661	0.01132
27	0.01024	0.00661	0.01128
28	0.01018	0.00660	0.01124
29	0.01011	0.00659	0.01120
30	0.01004	0.00659	0.01115
31	0.00997	0.00658	0.01111
32	0.00990	0.00657	0.01106
33	0.00983	0.00656	0.01101
34	0.00975	0.00655	0.01096
35	0.00968	0.00654	0.01091
36	0.00960	0.00653	0.01086
37	0.00953	0.00652	0.01080
38	0.00945	0.00650	0.01074
39	0.00937	0.00649	0.01069
40	0.00929	0.00647	0.01063
41	0.00921	0.00646	0.01057
42	0.00912	0.00644	0.01050
43	0.00904	0.00642	0.01044
44	0.00895	0.00640	0.01037
45	0.00886	0.00637	0.01031
46	0.00878	0.00635	0.01024
47	0.00869	0.00633	0.01017
48	0.00859	0.00630	0.01009
49	0.00850	0.00627	0.01002
50	0.00841	0.00624	0.00994
51	0.00831	0.00621	0.00986
52	0.00821	0.00618	0.00978
53	0.00811	0.00614	0.00969
54	0.00801	0.00611	0.00960
55	0.00791	0.00607	0.00952
56	0.00781	0.00603	0.00942
57	0.00770	0.00598	0.00933
58	0.00760	0.00594	0.00923
59	0.00749	0.00589	0.00913
60	0.00738	0.00584	0.00903
61	0.00727	0.00579	0.00892
62	0.00715	0.00574	0.00881
63	0.00704	0.00568	0.00870
64	0.00692	0.00562	0.00859
65	0.00680	0.00556	0.00847
66	0.00668	0.00550	0.00835
67	0.00656	0.00543	0.00823
68	0.00644	0.00537	0.00811
69	0.00631	0.00530	0.00798
70	0.00619	0.00522	0.00785
71	0.00606	0.00515	0.00771
72	0.00593	0.00507	0.00757
73	0.00580	0.00499	0.00743
74	0.00567	0.00490	0.00729
75	0.00554	0.00482	0.00714
76	0.00540	0.00473	0.00699
77	0.00527	0.00464	0.00684
78	0.00513	0.00454	0.00668
79	0.00499	0.00444	0.00653
80	0.00485	0.00434	0.00636
81	0.00471	0.00424	0.00620
82	0.00457	0.00414	0.00604
83	0.00443	0.00403	0.00587
84	0.00429	0.00392	0.00570
85	0.00415	0.00381	0.00552
86	0.00401	0.00370	0.00535
87	0.00387	0.00359	0.00518
88	0.00372	0.00347	0.00500
89	0.00358	0.00335	0.00482
90	0.00344	0.00323	0.00464
91	0.00330	0.00311	0.00446
92	0.00316	0.00299	0.00428
93	0.00302	0.00287	0.00410
94	0.00288	0.00275	0.00392
95	0.00274	0.00262	0.00373
96	0.00260	0.00249	0.00355
97	0.00246	0.00237	0.00336
98	0.00231	0.00224	0.00317
99	0.00217	0.00210	0.00298
100	0.00203	0.00197	0.00279
101	0.00188	0.00183	0.00259
102	0.00174	0.00169	0.00239
103	0.00159	0.00155	0.00219
104	0.00142	0.00139	0.00197
105	0.00124	0.00122	0.00172
106	0.00096	0.00095	0.00133
107	0.00000	0.00000	0.00000