

Mortality Models for the Advanced Ages in Japan



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Summary



Japan has one of the fastest aging populations in the world.

We, actuaries, have developed various models for forecasting mortality rates.

One of the most important themes for actuaries is to verify which model will most appropriately apply to the mortality rates of aging populations.

This paper examines two models with the intention of clarifying the degree of goodness of fit for Japanese mortality rate.

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Introduction



From the past to the present, actuaries have attempted various types of mortality rate forecasts.

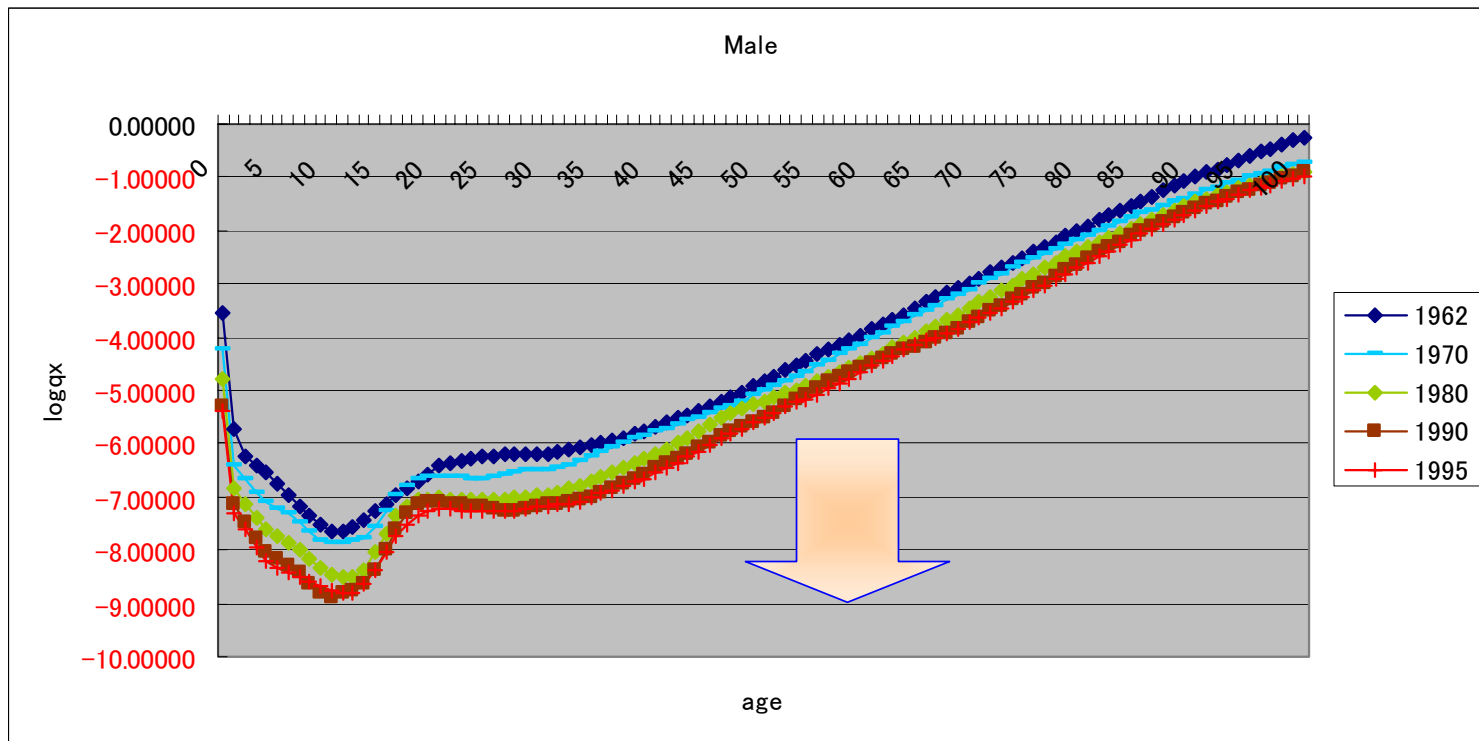
The purpose of this paper is to examine two types of mortality forecast models and evaluate fitness between the mortality rate forecast calculated using data from 1962 through 1995 and the experienced mortality rate for 1996 through 2004, in light of the coefficient of determination R^2 .

The History of Mortality Models

Author	Publication	Model
De Moivre	1725	$\mu_x = \frac{1}{\omega - x}$
Gompertz	1825	$\mu_x = BC^x$
Makeham	1860	$\mu_x = A + BC^x, \mu_x = \alpha + \gamma x + \beta c^x$
Opperman	1870	$\mu_x = \frac{a}{\sqrt{x}} + b + c\sqrt{x}$
Thiele	1872	$\mu_x = a_1 e^{-b_1 x} + a_2 e^{-\frac{1}{2} b_2 (x-c)^2} + a_3 e^{b_3 x}$
Wittstein	1883	$q_x = \frac{1}{m} a^{-(mx)^n} + a^{-(M-x)^n}$
Steffenson	1930	$\log_{10} l_x = 10^{-A\sqrt{x}-B} + C,$ $e_x = \frac{1}{A + Bc^x}$
...

The Methodology of Mortality Models

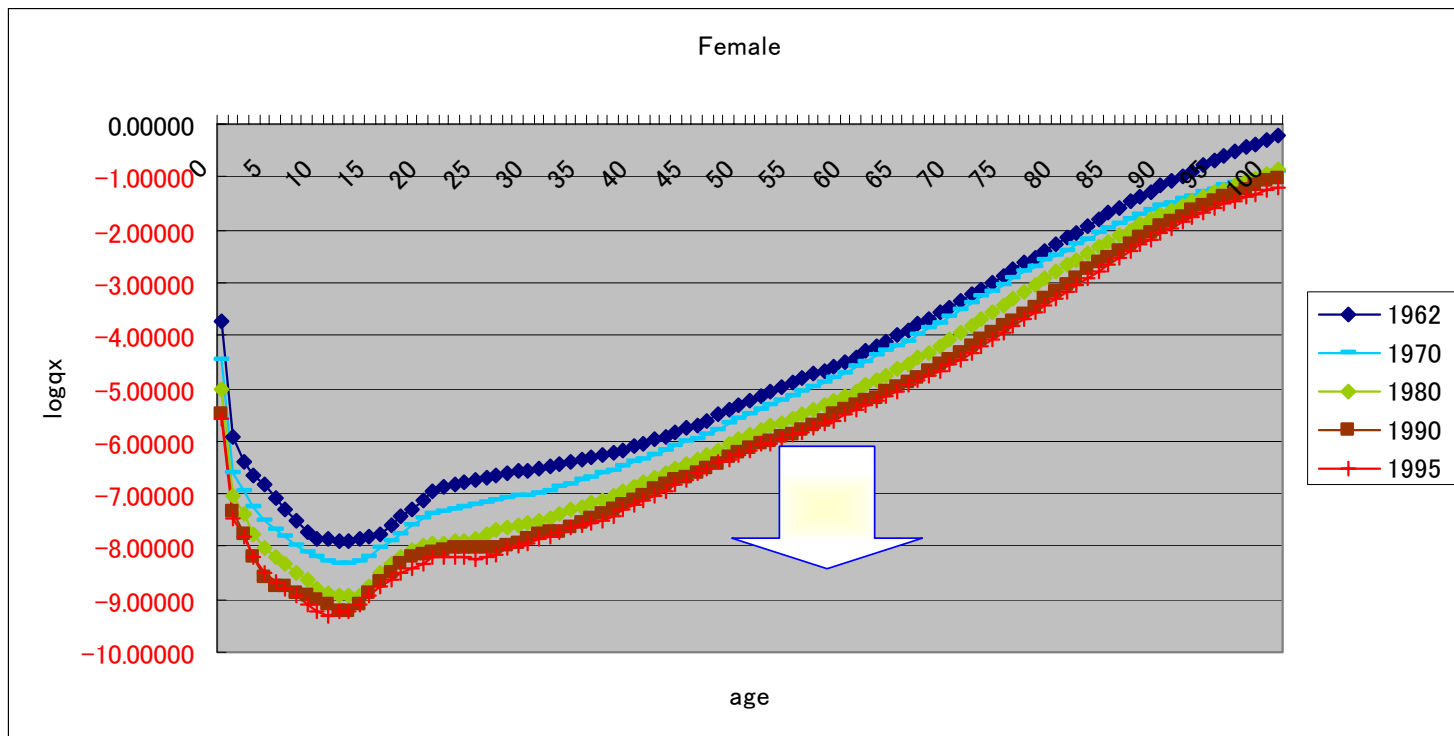
Data Used(Male)



The graph shows an improving trend in the mortality rate year by year.

The Methodology of Mortality Models

Data Used(Female)



The Methodology of Mortality Models

Data Used(2)

- In terms of the Lee–Carter Model for this paper, data from 1962 through 1995 were used to forecast the future mortality rate.
- With regard to the mortality rate forecast based on the method of the Japan Standard Mortality Table 2007 (Annuity mortality Table), the Japan Life Table 17 (based on the National Census of 1990) and the Vital Statistics of Japan (1970, 1978 and 1990) were used.

The Methodology of Mortality Models

Mortality Forecasting by Lee-Carter Model

- $\log(m_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t}$
 - x (age) and t (calendar year)
- $m_{x,t}$ is the observed central mortality rate at age x in year t
- a_x describes the average shape of the age profile
- b_x describes the pattern of deviations from the age profile as the k_t varies
- k_t describes the change in overall mortality ($\sum b_x = 1$ and $\sum k_t = 0$)
- $\varepsilon_{x,t}$ is the residual term at age x and time t
- a_x equal to the averages of the $\log(m_{x,t})$
- to get b_x and k_t from the first term of a singular value decomposition of the matrix:
$$R = (\log(m_{x,t}) - a_x)$$

The Methodology of Mortality Models

Mortality Forecasting by Lee-Carter Model (2)

• The National Institute of Population and Social Security Research, in the estimate from 2002, expressed its view that it will make a more natural assumption that the mortality rate will show a gradually improving trend rather than to assume that the mortality rate will continue to improve in the future as in the past at a pace greater than other advanced countries, in view of the fact that Japanese mortality rate improved dramatically after World War II and has already reached the world's highest level.

• We used a function to estimate k_t

$$k_t = [\alpha_1 + \alpha_2 \exp\{(t + \alpha_4) / \alpha_3\} + \beta_1 + \beta_2 \log(t + \beta_3)] / 2$$

The Methodology of Mortality Models

Mortality Forecasting by Japan Standard Mortality Table 2007(JSMT2007)

- (1) As a basic mortality rate, the Japan Life Table 19 (based on the National Census of 2000) shall be used.
- (2) Calculate the declining rate (improvement rate) per annum of mortality rate by gender and in five-year age brackets using the improvements in mortality rates by gender by each five-year age bracket and by cause of death from 1980 through 2000.
 - (2-a) Calculate the annual average improvement rate for 1980 through 2000 by gender, by each five-year age bracket, and by 8 causes of death, according to the Vital Statistics of Japan.
 - (2-b) By using the annual average improvement rate by cause of death reached by (2-a) above, the future mortality rate of the median age for each five-year age bracket was forecast by cause of death. The representative year of birth for forecasting purposes was 1960. In addition, regarding the cause of death, the average past improvement rate for which has been negative, then the future improvement rate shall be set to zero.

The Methodology of Mortality Models

Mortality Forecasting by Japan Standard Mortality Table 2007(JSMT2007)(2)

- (2-c) Calculate the annual average improvement rate of the total causes of death for the median ages using the total value of mortality rates by cause of death calculated by (2-b) above as well as the mortality rate for 2000 (sum of causes of death), and, through the method of linear interpolation between ages, calculate the annual average improvement rate for each age.
- (3) Based on the understanding that the mortality rate will continue to improve every year at the improvement rate calculated in (2) above, the future mortality rate shall be estimated. The "future" to be estimated shall, in principle, be the year when a person born in 1960 reaches each age, and the mortality rate of the Japan Life Table 19, to which the improvement was made considering the mortality rate corresponding to the number of years from 2000, shall be "the future mortality rate." (The improvement in mortality rate over at least 20 years shall be taken into account.)
- (4) The future mortality rate reached by (3) above, extrapolated in terms of advanced ages and young ages, as well as further adjusted in the direction of survival risk, shall be deemed the mortality rate on the Annuity Mortality Table.

The Methodology of Mortality Models

Mortality Forecasting by Japan Standard Mortality Table 2007(JSMT2007)(3)

In this paper, the mortality rate based on this method has been calculated by using the data available in 1995, that is to say, the Japan Life Table 17 and the Vital Statistics of Japan (1970, 1978 and 1990).

In addition, this life table is not intended for liability valuation but for mortality rate forecasts for 1996 onward; therefore, setting the improvement rate by cause of death at no more than zero as stated in (2-b) above as well as adjustment in the direction of survival risk stated in (4) above were not implemented.

The Methodology of Mortality Models

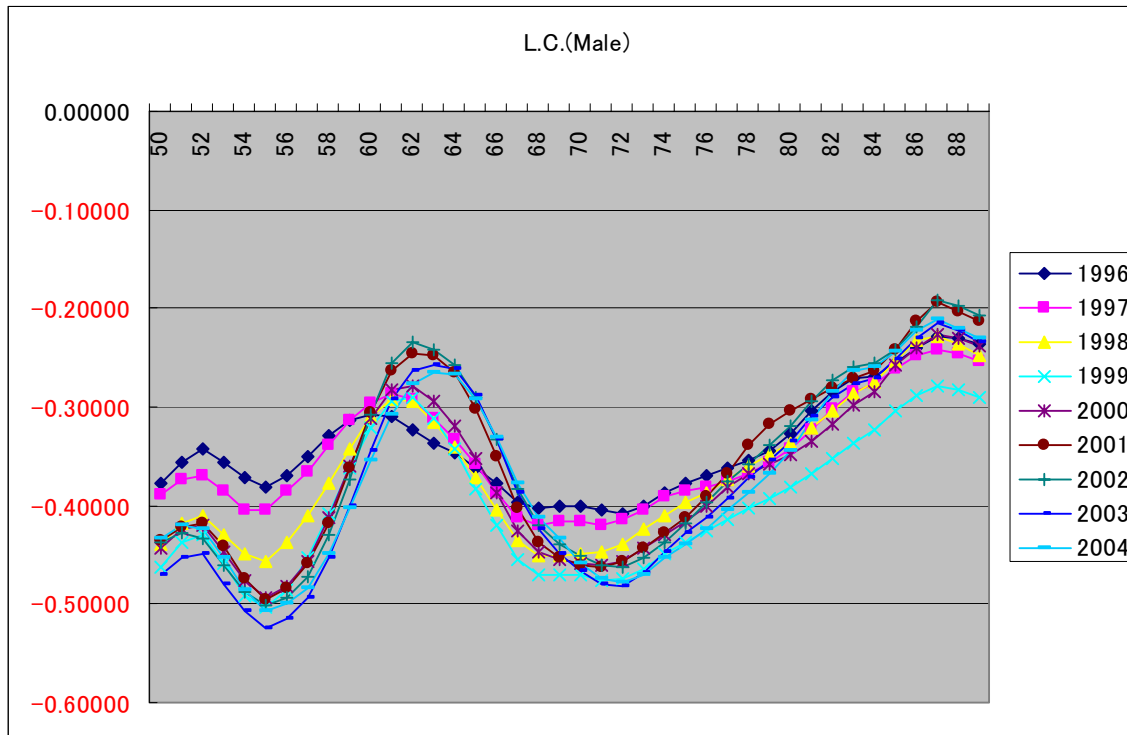
Method of Comparison between models

The mortality rate forecast for 1996 through 2030 was calculated using the Lee-Carter Model method, and the mortality rate forecast for 1996 onward was implemented by means of the JSMT2007 method.

	Data used	By cause of death or otherwise	Parameters/method used for forecasting for the future
Lee-Carter Model	Abridged life table and the complete life table for Japan (1962-1995)	Regardless of cause of death, the mortality rate shall be forecast based on the death as a whole	In order to forecast k_t , application of functions has been implemented.
JSMT2007	Japan Life Table 17 and Vital Statistics of Japan (1970, 1978 and 1990)	Forecast the improvement rate by cause of death.	Setting of the improvement rate by cause of death at no more than zero and the adjustment in the direction of survival risk were not implemented.

The Result

- The result of comparison of the difference in logarithmic values of mortality rates — Lee-Carter Model(Male)

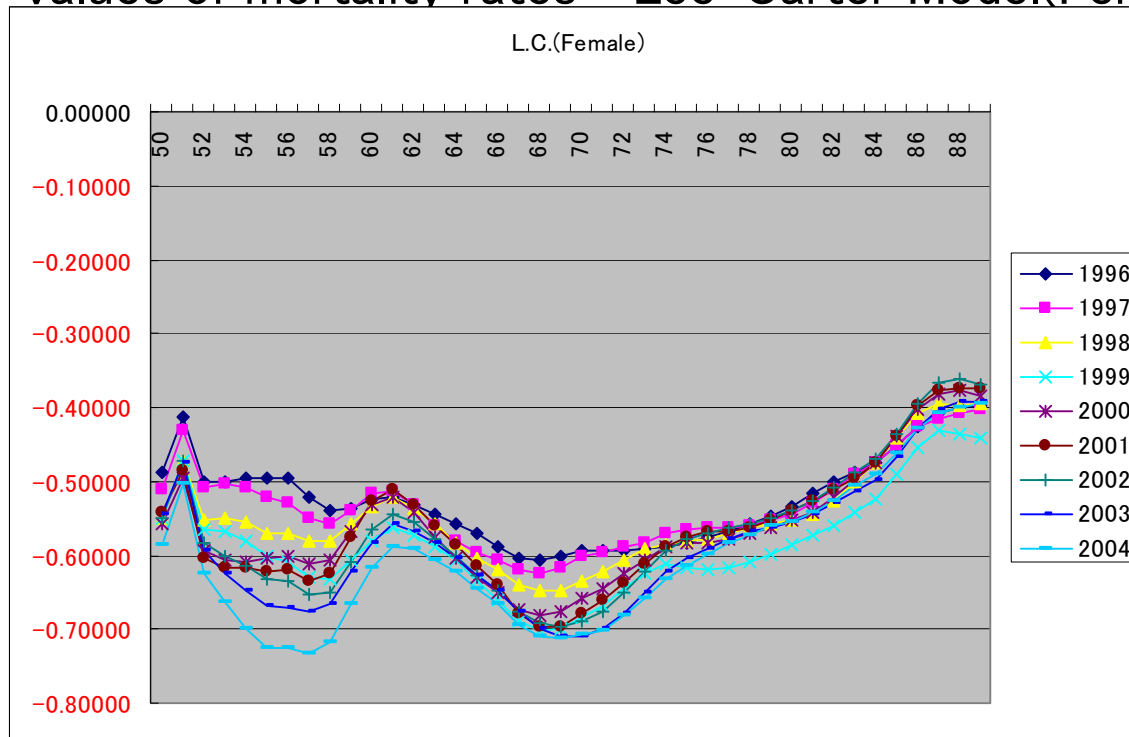


log”Mortality rate
forecast”

–log”Experienced
mortality rate”

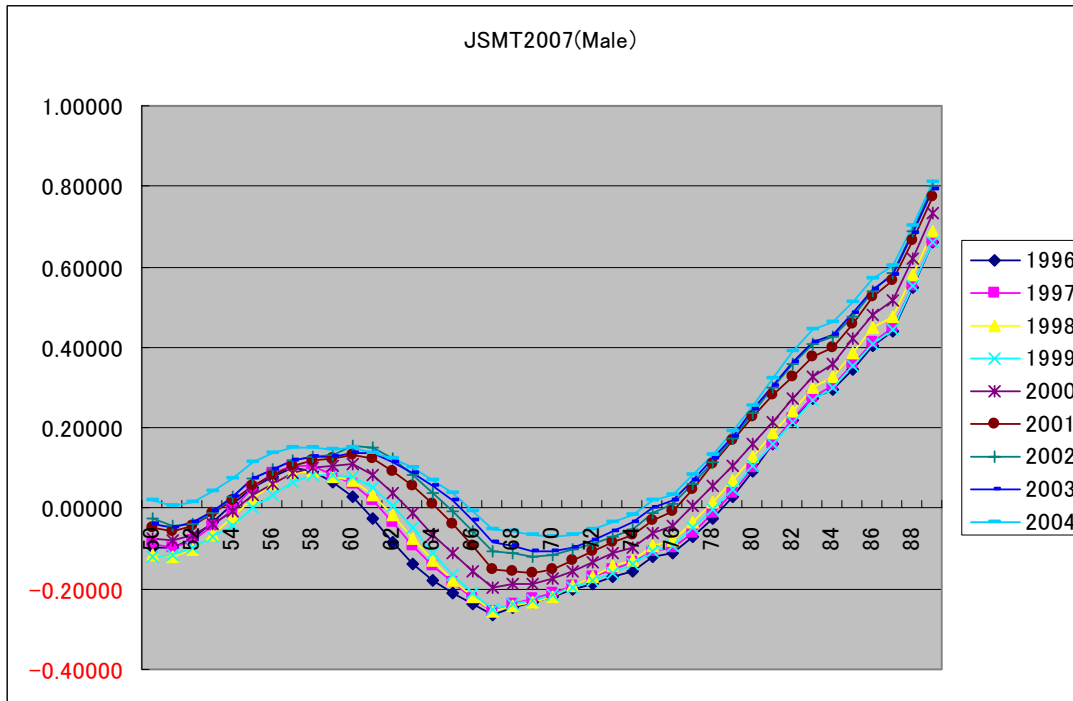
The Result

- The result of comparison of the difference in logarithmic values of mortality rates — Lee-Carter Model(Female)



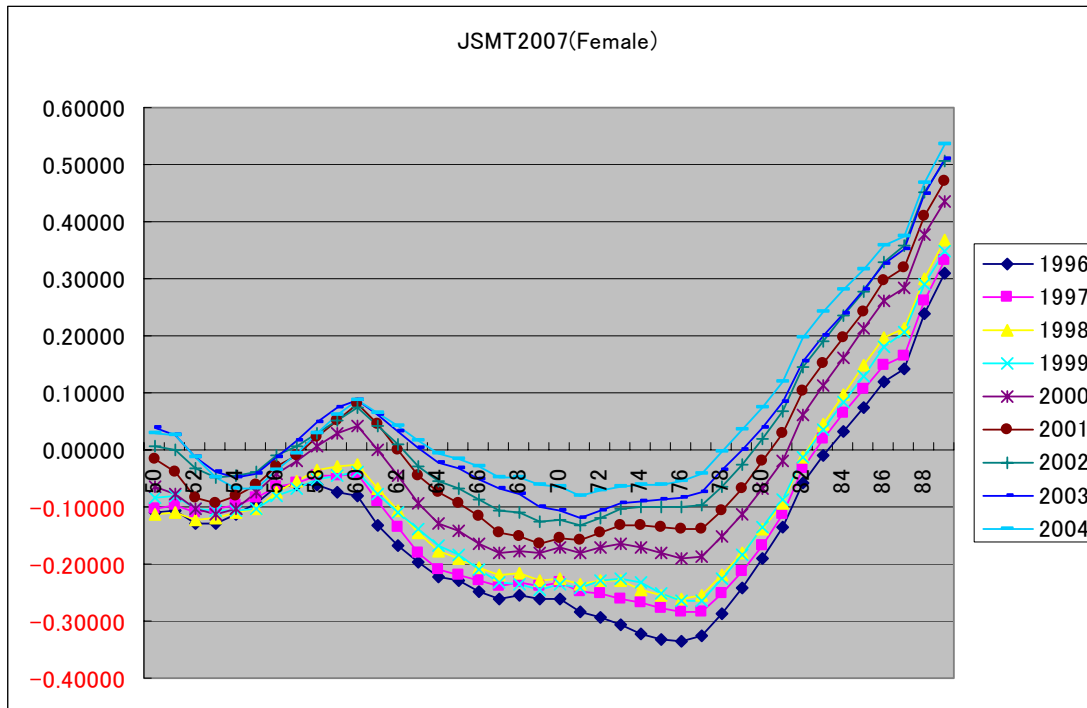
The Result

- The result of comparison of the difference in logarithmic values of mortality rates(2) — JSMT2007(Male)



The Result

- The result of comparison of the difference in logarithmic values of mortality rates(2) — JSMT2007(Female)

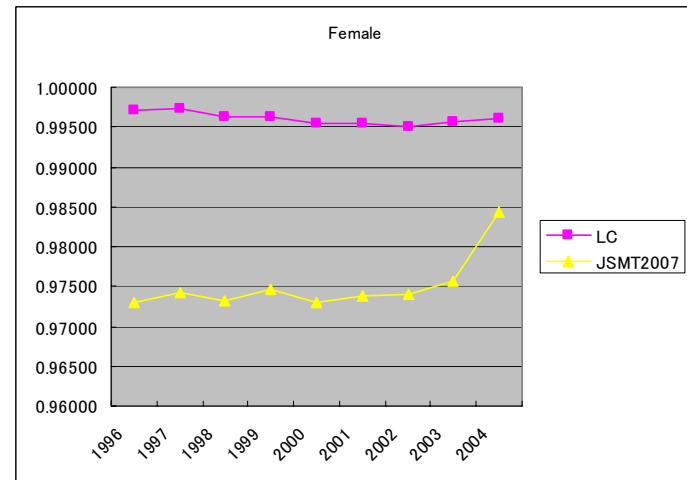
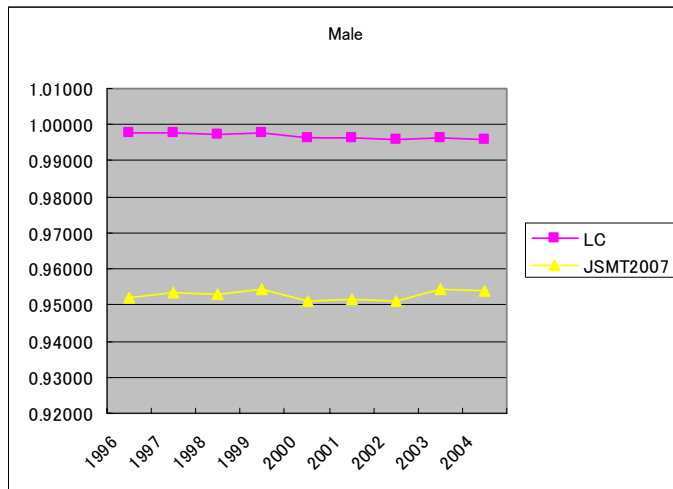


The Result

- The result of comparison of the difference in logarithmic values of mortality rates — JSMT (3)
 - The mortality rate forecast by means of the Lee–Carter Method proved to be lower than the experienced mortality rates. The mortality rate forecasts by means of JSMT 2007 are, in general, lower than the experienced mortality rates up to the late 70s and are higher than the experienced mortality rates thereafter.
 - In the age brackets of 80 and above for the JSMT 2007, the range of deviation from the experienced mortality rates in the direction of worsening mortality rates expands gradually along with advancing age. Such deviation of the forecast rates in the direction of worsening mortality rates is considered to have been caused because the setting of the improvement rate by cause of death at no more than zero, which is adopted on JSMT2007 in order to estimate the mortality rate improvement in a conservative manner, is not implemented in this paper.

The Result

—The R^2 of all ages(0 to 89 years old) by type of method



The Result



- The R^2 of all ages (0 to 89 years old) by type of method (2)
 - Across all age brackets, from 1996 through 2004, the fitness of the JSMT 2007, in terms of both men and women, keeps improving, which leads us to consider that the JSMT 2007 is suitable for forecasting mortality rates for the long term.

The Result

— The method which shows better fitness

Male									
R^2	1996	1997	1998	1999	2000	2001	2002	2003	2004
0-89	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.
50-54	JSMT2007	L.C.	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007
55-59	L.C.	L.C.	L.C.	L.C.	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007
60-64	L.C.	L.C.	L.C.	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007
65-69	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.
70-74	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007
75-79	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.
80-84	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.
85-89	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.

Female									
R^2	1996	1997	1998	1999	2000	2001	2002	2003	2004
0-89	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.
50-54	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007
55-59	L.C.	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007
60-64	L.C.	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007
65-69	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007	JSMT2007
70-74	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	JSMT2007	JSMT2007
75-79	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.
80-84	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.
85-89	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.	L.C.

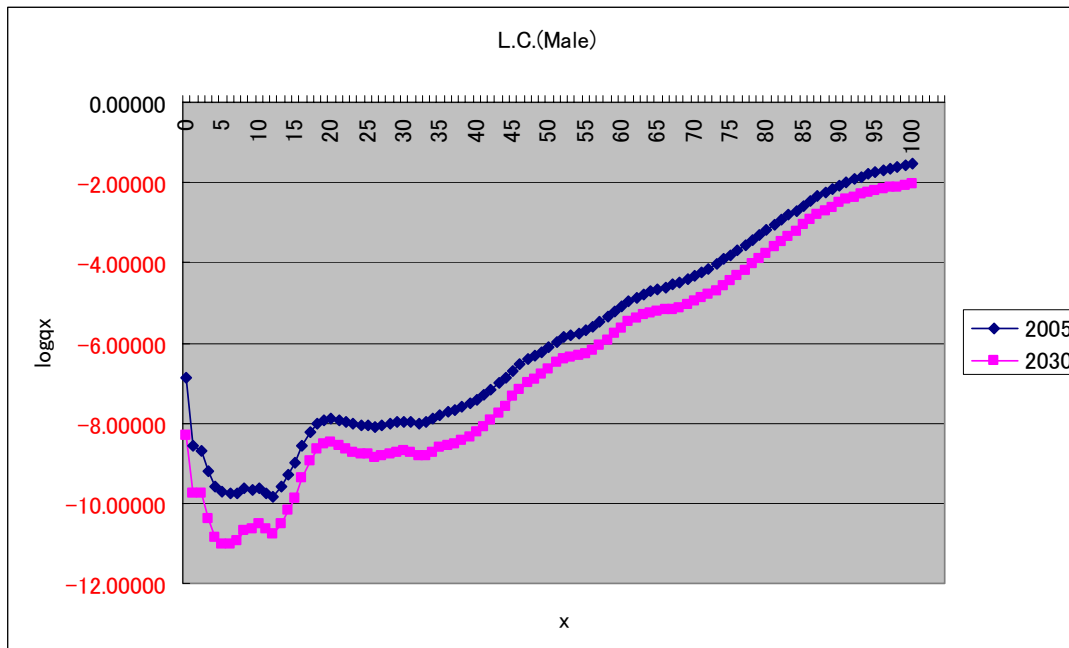
The Result



- The method which shows better fitness(2)
 - After comparing the fitness for the experienced mortality rates of 1996 through 2004, the Lee–Carter Model showed the best fitness for both men and women over all ages.
 - During the period from 1996 through 2004, over time, the ages where the JSMT 2007 shows better fitness than the Lee–Carter Model tend to increase. This also implies that the JSMT2007 is a method suitable for forecasting mortality rates for the long term.

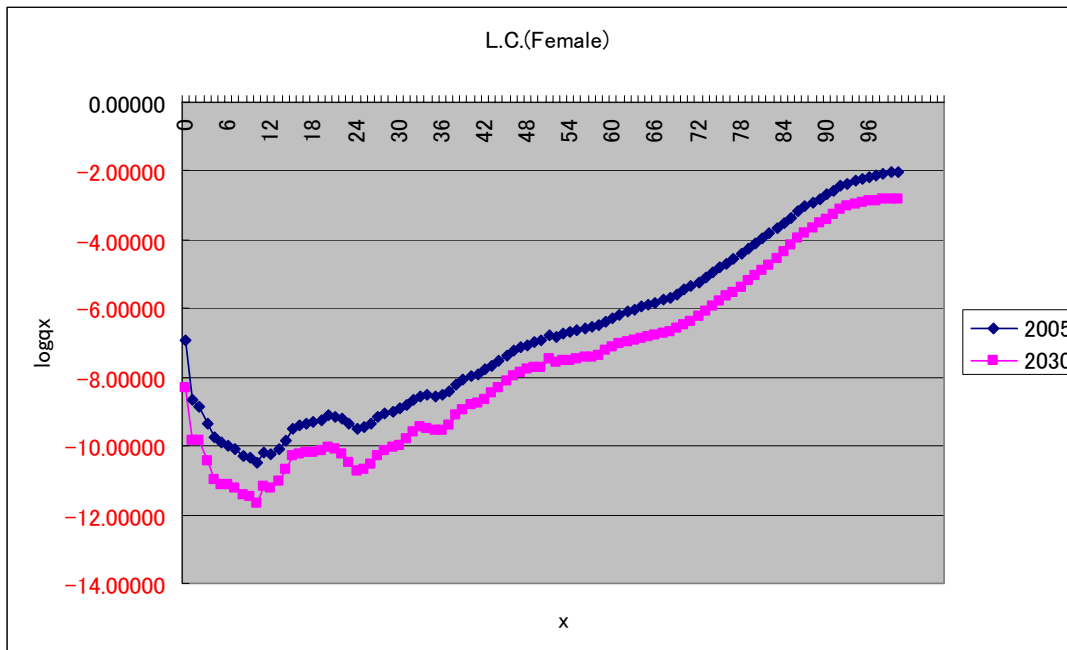
The Result

— Comment on the graph of the future mortality rate(Male)



The Result

— Comment on the graph of the future mortality rate(Female)



It has turned out that the improvement value is estimated around 90‰ at maximum.