Dilution effects of executive stock option awards

by
P. W. A. Dayananda
and J. T. Kemper,
University of St.Thomas,2115
Summit Ave,MN 55105,USA

OUTLINE

- Describe the characteristics of Executive Stock Options (ESO).
- Assumption about stock price process.
- Describe how the dilution of shares occur when ESOs are exercised.
- Describe the Approximation carried out.
- Derive the expected Dilution.
- Sensitivity of Dilution & Numerical illstration.

Characteristics of ESO

These are CALL options exercisable at time at a time in the future by paying an exercise price & has following main special features:

- Vesting period
- Non-transferability
- If the executive leaves the firm before exercise, it has no value

Stock Price Process/GRANT

- Assume that the stock price at time t, $S(t) = S(0)\exp\{X(t)\}$ where X(t) is normal with mean μt and variance $\sigma^2 t$
- Suppose that the ESO grant consists of a series exercisable at $\tau_1, \tau_2, \ldots \tau_m$ with respective exercise prices $K_1, K_2, \ldots, K_m$;respective numbers been

$n_1, n_2, \ldots, n_m$

How Dilution Occurs ?

- Let $X(\tau_i) = X_i, i = 1,2,..,m$. Consider dilution that would occur at time τ_1 .
- When an ESO is exercised, the net additional shares to be issued is $1 - \frac{K_1}{S(0)} e^{-X_1}$
- Let $\ln(K_i / S(0)) = k_i$, N_i be the number of shares and the respective Dilution at time
- Let $A_i = \{X_i \le k_i\}$
- Then Dilution is $D_1 = \frac{I(A_1)}{N_0} + \frac{I(\overline{A}_1)}{[N_0 + n_1(1 - e^{-X_1 + k_1})]}$ (1)

Expectation of Dilution

- Neglect terms of order $(\frac{n_1}{N_0 + n_1})^2$ and taking risk-neutral expectation of (1), we get

$$E_Q(D_1) = E_Q(D_0) - \frac{n_1}{N_0(N_0 + n_1)} E_Q[I(X_1 > k_1) + \frac{n_1}{(N_0 + n_1)^2} E_Q[e^{-X_1} I(X_1 > k_1)] \qquad (2)$$

- Now $E_Q[I(X_1 > k_1)] = E[I(X_1 > k_1; h]$ where h is the Esscher transform parameter (Gerber & Shiu,1994)

Esscher Transform

- Under Esscher Transform (risk-neutral measure) , X_1 has mean per unit time $\mu + h\sigma^2 = r - \sigma^2/2$ and variance σ^2 . σ^2

- Hence $E[I(X_1 > k_1; h] = \Phi(-\frac{k_1 - (r - \sigma^2/2)\tau_1}{\sigma\sqrt{\tau}})$ (3)

where $\Phi(.)$ denotes
the distribution function of the standard normal random variable.

Expected value of Dilution at τ_1

- Using the result from (3) and substituting in (2), we have the expected value of dilution at the first time of exercise as

$$E_Q(D_1) = E(D_0) - \frac{n_1}{N_0(N_0+n_1)}\Phi\left(-\frac{k_1-(r-\sigma^2/2)\tau_1}{\sigma\sqrt{\tau_1}}\right)$$

$$+\frac{n_1 K_1}{S(0)(N_0+n_1)^2} e^{(\sigma^2-r)\tau_1}\Phi\left(-\frac{k_1-(r-3\sigma^2/2)\tau_1}{\sigma\sqrt{\tau_1}}\right). \qquad (4)$$

Generalization of Dilution

- In order to generalize the expression for the expected value of Dilution we first note that

$$N_i = N_{i-1} + n_i(1 - \frac{K_i}{S(0)} e^{-X_i}) \qquad , X_i > k_i$$

$$N_i = N_{i-1}$$

General Expression for Dilution

$$E_Q(D_m) = \frac{1}{N_0} - \sum_{i=1}^{m} \frac{n_i}{(N_0 + \sum_{j=1}^{i-1} n_j)(N_0 + \sum_{j=1}^{i} n_j)} (\Phi(d_1(i)) + \frac{1}{S(0)} \sum_{i=1}^{m} \frac{n_i K_i e^{(\sigma^2 - r)\tau_i}}{(N_0 + \sum_{j=1}^{i} n_j)^2} \frac{K_i e^{(\sigma^2 - r)\tau_i}}{S(0)} \Phi(d_2(i)),$$

$$d_1(i) = \frac{(r - \sigma^2 / 2)\tau_i - k_i}{\sigma\sqrt{\tau_i}}$$

$$d_2(i) = d_1(i) - \sigma\sqrt{\tau_i}$$

Sensitivity to Dilution

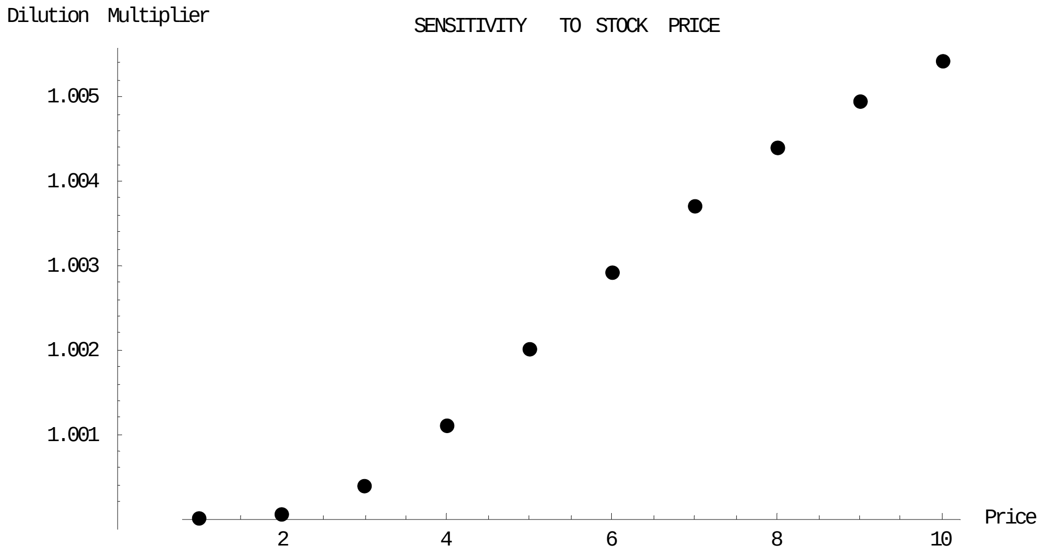
- The sensitivities of dilution are obtained by deriving the derivatives. These are when m=1:

$$\frac{\partial}{\partial S(0)} E_Q(D_1) = -\frac{n_1 K_1 e^{(\sigma^2 - r)\tau_1}}{S(0)^2 (N_0 + n_1)^2} \Phi(d_2(1))$$

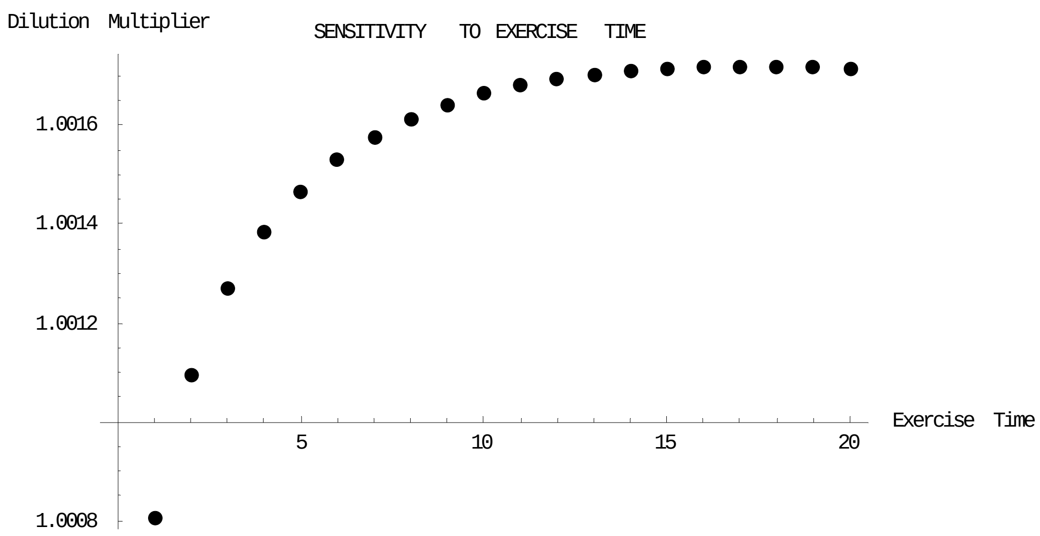
$$\frac{\partial}{\partial \tau_1} E_Q(D_1) = \frac{n_1 K_1 (\sigma^2 - r) e^{(\sigma^2 - r)\tau_1}}{S(0)^2 (N_0 + n_1)^2} \Phi(d_2(1))$$

$$-\frac{n_1 \sigma \phi(d_1(1))}{2(N_0 + n_1)^2 \sqrt{\tau_1}} - \frac{n_1^{\,2} \phi(d_1(1))}{2N_0 (N_0 + n_1)^2} [(\frac{r}{\sigma} - \frac{\sigma}{2})\tau_1^{-.5} + \frac{k_1}{\sigma} \tau_1^{-1.5}].$$

Numerical Illustration-1



Numerical Illustration-2



Numerical Illustration-3